

# ***SOME COSMOLOGICAL MODELS THEIR TIME SCALES AND SPACE METRICS***

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## ***1. Introduction***

Accepting clock retardation as an empirical fact, we provisionally adopt Whitrow's derivation of the Robertson-Walker Metric (**RWM**) of Cosmology from the  $\gamma$ -factor of Special Relativity (**SR**). According to Milne, and Walker, two kinds of *observers* must be distinguished: *fundamental* ones which define the geometrical structure of the specific cosmological model under consideration by constituting its substratum, and *accidental* ones which are superposed on the substratum in a way that refers the description of their motion to the *substratum* as a universal "frame of rest". The difference, of course, is statistical and a matter of degree. It is therefore possible to make sense of a graduation of clocks according to their approximation to the ideal of a universal time: we classify particles by estimating the departure of their distribution from universal isotropy. Recalling the fact that the principle of *cosmic isotropy* can be used as an argument for the definability of an all-embracing *universal time*, at least statistically, we propose to reverse this procedure by postulating such time as a *regulative idea* in the sense of Kant.

Taking **RWM** as our formal point of departure we next investigate the properties of two standard models of modern cosmology:  $\alpha$ ) the uniform expansion model of Milne & Prokhorovnik, which is the simplest model of a cosmic "big bang", and  $\beta$ ) the exponential expansion model of Bondi & Gold, supposed to be the simplest model of a cosmic "steady state". Rejecting the so-called "perfect cosmological principle" of the latter, it is easy to show that the ideas of "big bang" and "steady state" may not be mutually exclusive after all: a universe starting with a "big bang" at the dawn of creation may very well approximate to a "steady state" in the course of infinite time. In agreement with our preceding analysis of **RWM** we further consider the relationship between our choice of time scale for a particular model of the universe and its corresponding space metric. As it turns out, there are at least two important ways of mapping the expansion of the universe: a) that which keeps atomic sizes constant while light is being stretched, and b) that which keeps distances between fundamental particles constant while their constituents are shrinking.

Finally a new model of the universe is proposed which deviates from **RWM** by allowing the curvature of space to vary with distance. In this model the curvature of space appears to increase with the distance at which an object is observed by a fundamental observer. The model suggested is a new "steady state" model which is even simpler than that of Bondi & Gold in the sense that it implies an absolute structural identity between "world map" and "world view" (Milne's concepts). The basic properties of this model and related ones are examined and discussed.

## 2. The Lorentz Transformations

Given  $c \equiv \text{unity}$ ,  $v \equiv \tanh\omega$ , the **LT** are expressible in purely temporal coordinates:

$$\begin{aligned} t' &= t \cosh\omega - x \sinh\omega \\ x' &= x \cosh\omega - t \sinh\omega \end{aligned}$$

The hyperbolic standard formulae corresponding to the simple (Galilean) addition  $\alpha' \equiv \alpha - \omega$  are:

$$\begin{aligned} \cosh\alpha' &= \cosh\alpha \cosh\omega - \sinh\alpha \sinh\omega \\ \sinh\alpha' &= \sinh\alpha \cosh\omega - \cosh\alpha \sinh\omega \end{aligned}$$

It is interesting to see that **LT** are compatible to the addition formulae if, and only if,  $\mathcal{T} \equiv \mathcal{T}'$  in:

$$\begin{aligned} t'/\mathcal{T}' &\equiv \cosh\alpha' , \quad t/\mathcal{T} \equiv \cosh\alpha \\ x'/\mathcal{T}' &\equiv \sinh\alpha' , \quad x/\mathcal{T} \equiv \sinh\alpha \end{aligned}$$

It appears natural to identify  $\mathcal{T}^2$  with the **SR** invariant:  $\mathcal{T}^2 \equiv t^2 - x^2 - y^2 - z^2 = t'^2 - x'^2 - y'^2 - z'^2$ .

The standard (1x3) **LT** for three inertial frames,  $\Sigma$ ,  $S$  &  $S'$ , in relative motion are:

$$\begin{aligned} T &\equiv t \cosh\alpha - x \sinh\alpha = t' \cosh\alpha' - x' \sinh\alpha' , \quad Y \equiv y \equiv y' \\ X &\equiv x \cosh\alpha - t \sinh\alpha = x' \cosh\alpha' - t' \sinh\alpha' , \quad Z \equiv z \equiv z' \end{aligned}$$

Consider  $\Sigma$  to be a preferred frame with the privileged observer  $\Pi$  placed in its *origo*. Let the observers  $P$  &  $P'$  be situated in the *origos* of  $S$  &  $S'$ , resp., and let the frame times  $t$  &  $t'$  of  $S$  &  $S'$  be synchronized to the proper time  $T$  of  $\Pi$  by choosing  $T \equiv t \equiv t' \equiv 0$  when  $P$  &  $P'$  both coincide with  $\Pi$ . Suppose that an event  $E$  happens at object  $O$ , as perceived by  $\Pi$ ,  $P$  &  $P'$ . Let the standard coordinates of  $E$  be  $(T, X, Y, Z)$  in  $\Sigma$ ,  $(t, x, y, z)$  in  $S$ , and  $(t', x', y', z')$  in  $S'$ . Then, by eliminating the irrelevant frame times  $t$  &  $t'$  from the expressions for  $T$  &  $X$ , we obtain:

$$X = x/\cosh\alpha - T \tanh\alpha = x'/\cosh\alpha' - T \tanh\alpha'$$

Further, using  $\omega = \alpha - \alpha' = -\omega'$  to eliminate  $\alpha$  or  $\alpha'$ , we recover **LT** for the privileged time  $T$ :

$$\boxed{\begin{aligned} x' &= \{x \cosh(\omega - \alpha) - T \sinh\omega\} / \cosh\alpha \\ x &= \{x' \cosh(\omega' - \alpha') - T \sinh\omega'\} / \cosh\alpha' \end{aligned}}$$

Finally, introducing non-standard frame-times  $\tau$  &  $\tau'$  for the frames  $S$  &  $S'$  defined by means of

$$\boxed{T \equiv \tau \gamma \equiv \tau' \gamma'}$$

$$\gamma \equiv \cosh\alpha \equiv 1/\sqrt{1-v^2}, \quad \gamma' \equiv \cosh\alpha' \equiv 1/\sqrt{1-v'^2}$$

and using  $w \equiv \tanh\omega$ , we recover the Tangherlini transformations (**TT**) generalized by Selleri:<sup>1</sup>

$$\begin{aligned} x' &= x \frac{\cosh(\omega - \alpha)}{\cosh\alpha} - \tau \sinh\omega = \frac{x(1-wv) - w\tau}{\sqrt{1-w^2}} \\ x &= x' \frac{\cosh(\omega' - \alpha')}{\cosh\alpha'} - \tau' \sinh\omega' = \frac{x'(1-w'v') - w'\tau'}{\sqrt{1-w'^2}} \end{aligned}$$

In standard **SR**, it is always the proper time of a single moving clock which is said to be retarded relative to the slave clocks distributed as a network over the rest frame of the observer; but if we refer the inertial motion of particles to some privileged frame we should use **TT** instead. It is worth noticing that **TT** reduce to **GT** if all measurements are referred to the "midway frame":

$$\boxed{\alpha = \frac{\omega}{2} \Rightarrow x - x' = \tau \sinh\omega = 2T \sinh\frac{\omega}{2}}$$

Inserting  $\tau \equiv t - x \tanh\frac{\omega}{2} \equiv t' - x' \tanh\frac{\omega'}{2}$  directly into **LT** we get the same result, viz. **GT**:<sup>2</sup>

$$\boxed{\tau' = \tau, \quad \omega' = -\omega, \quad x' = x - \tau \sinh\omega, \quad y' = y, \quad z' = z}$$

### 3. The Robertson-Walker Metric

In his monumental *Natural Philosophy of Time* (1961/1980),<sup>3</sup> G.J. Whitrow sketched a method to derive the **RWM** of relativistic standard cosmology from the  $\gamma$ -factor of **SR**. For simplicity, put:

$$\boxed{c_o \equiv T_o \equiv R_o \equiv 1}$$

Then define the *origo* of the comoving standard rest frame  $S$  of an observer  $P$  to be  $P$  himself. Now suppose an event  $E$ , taking place at some object  $O$ , to be triggered by a light signal which instantaneously released a visible flash. Suppose further that this light signal was sent off by  $P$  at the instant  $\hat{t}_1$ , and that the flash was perceived by  $P$  at the instant  $\hat{t}_3$ , both  $\hat{t}_1$  &  $\hat{t}_3$  being instants of the proper time of  $P$  as read off his own standard atomic clock  $C$ . Using  $d$  to denote differentials of proper time  $\hat{t}$ , and  $\delta$  to denote differentials of frame time  $t$  and frame distance  $r$ , we construe the Einstein coordinates of the Cartesian frame  $S$  of  $P$  by means of the definitions:

$$t \equiv \frac{1}{2}(\hat{t}_3 + \hat{t}_1), \quad r \equiv \frac{1}{2}(\hat{t}_3 - \hat{t}_1)$$

From the standard **SR** invariant we then get the  $\gamma$ -factor for the retardation of "moving clocks":

$$d\mathcal{T}^2 \equiv d\hat{t}_3 d\hat{t}_1 = \delta t^2 - \delta r^2 \equiv \gamma^{-2} \delta t^2$$

Whitrow now suggested the substitutions:  $\delta t \rightarrow dT$ ,  $\delta r \rightarrow \mathcal{E}(T) \delta\sigma$ . In this,  $T$  is no longer the time of an inertial frame, but the common **Cosmic Time** of all fundamental observers, i.e. all observers "at rest" in the universe, for instance relative to **CBR**, the cosmic background radiation; further,  $\mathcal{E}(T)$  represents the expansion of the universe, while  $\sigma$  is a fixed (comoving) coordinate. This transforms the standard invariant of **SR** into the standard metric of modern cosmology:

$$\boxed{d\mathcal{T}^2 = \delta T^2 - \mathcal{E}^2(T) \delta\sigma^2}$$

What Whitrow has done is thus to propose a reinterpretation of the traditional invariant of **SR**. However, one can legitimately query whether this amounts to anything more than mere analogy. Are not the involved concepts changed beyond recognition? But let us accept it at face value!

For *fundamental observers*  $\delta\sigma = 0$ , or  $d\mathcal{T} = dT$ , showing that all fundamental observers participate in the same absolute or universal time  $\mathcal{T}$ . Per implication, any deviation from  $\mathcal{T}$  of proper time  $T$  is restricted to non-fundamental or *accidental observers* marked out by variable  $\sigma$ . According to the usual interpretation of **SR**, it is always the *proper time* of a "moving" particle which is said to be "slow" relative to the *frame time* of a "stationary" observer, and frame time, or coordinate time, is thereby tacitly assumed to represent the "true extended time" of an observer. On the other hand, even though the cosmic time implied by **RWM** is seldom taken seriously, but mostly rather neglected or explained away as being of "statistical origin", hence "ill defined", it is the firm stance of the present writer that fundamental importance should be ascribed to it.

If we define *true time* by the readings of the master clocks of the fundamental observers - i.e. when they have been properly synchronized, e.g. by letting a definite non-local cosmic event, such as the beginning of everything in a so-called "big bang", represent their common time zero - then it is clearly not true to say that the clocks of fundamental particles are "slow" relative to the frame time of some non-fundamental observer particle; much rather it is true to say that it is the clocks of accidental particles which are "slow" relative to the clocks of the universal substratum. However, everything depends on convention in the sense that it follows from our point of view. But, please, notice that this does not involve us in any conflict with the results of experiment, since the only conflict at stake is one relating to the interpretation of established theory!

Hence, if the clocks of fundamental observers show the true time, then the clock of an accidental particle will be more or less astray. In fact, the greater its distance to that fundamental particle relative to which it is momentarily at rest, and which thereby constitutes the natural origo of its own rest frame, the slower its clock will run and the more it will deviate from true time. The natural way of interpreting this slowing down of moving clocks is as an effect of gravitation. In this way we have found a natural coupling between the rates of non-fundamental clocks and what seems to be a gravitational potential due to the substratum of fundamental particles!

The reason for the dependence disclosed is that the deviation of the clock of an accidental particle from true time is found by direct comparison with the clock of that fundamental particle with which it momentarily coincides; and the greater the distance of an accidental particle is to the origo of that rest frame to which it belongs, the faster its speed relative to that fundamental particle with which it coincides will appear; this is a direct consequence of the expansion function  $\mathcal{E}(T)$ . What we have disclosed is the possibility of an influence of the substratum on particles which do not belong to the substratum and which thus represent deviations from cosmic symmetry.

This supports a conjecture of Whitrow's former master, E.A. Milne. The essential point of his *Kinematic Relativity*,<sup>4</sup> intended as an alternative to Einstein's *General Relativity*, is precisely that what we call gravitational effects may emerge from local deviations from cosmic symmetry. Indeed, when elevated to a universal principle, Milne's conjecture amounts to nothing less than a direct contradiction of Mach's: where Mach claimed that inertia should be reduced to gravitation, Milne instead held that gravitation should be reduced to inertia - and demonstrated how to do it! But all this is a repetition of ideas presented earlier. With polar coordinates the **RWM** becomes:

$$\boxed{d\mathbf{T}^2 = dT^2 - \mathcal{E}^2(T)\{\delta\rho^2 + \lambda^2(\delta\theta^2 + \sin^2\theta \delta\phi^2)\}}$$

$$R \equiv \mathcal{E}(T)\rho, \quad \mathbf{T} \equiv \int dT/\mathcal{E}(T) + const., \quad \mathcal{H}(\mathbf{T}) \equiv d\mathbf{T}/dT$$

$R$  is proper distance and  $\mathcal{H}$  is the Hubble function, while  $\mathbf{T}$  is an auxiliary time scale.

$$\delta\rho \equiv \delta\lambda/\sqrt{1-\kappa\lambda^2} = \begin{cases} \delta\lambda & \Leftarrow \kappa = 0 \\ \delta \arcsin\lambda & \Leftarrow \kappa = 1 \\ \delta \operatorname{arsinh}\lambda & \Leftarrow \kappa = -1 \end{cases}$$

$$\boxed{\underset{\kappa=0}{d\mathbf{T}^2} = dT^2 - \mathcal{E}^2(T)\{\delta\rho^2 + \rho^2(\delta\theta^2 + \sin^2\theta \delta\phi^2)\}}$$

$$\boxed{\underset{\kappa=1}{d\mathbf{T}^2} = dT^2 - \mathcal{E}^2(T)\{\delta\rho^2 + \sin^2\rho(\delta\theta^2 + \sin^2\theta \delta\phi^2)\}}$$

$$\boxed{\underset{\kappa=-1}{d\mathbf{T}^2} = dT^2 - \mathcal{E}^2(T)\{\delta\rho^2 + \sinh^2\rho(\delta\theta^2 + \sin^2\theta \delta\phi^2)\}}$$

$$\delta\rho \equiv \delta\lambda/\sqrt{1-\kappa\lambda^2} \equiv \delta\rho/(1+\kappa\rho^2/4) = \begin{cases} \delta\rho & \Leftarrow \kappa = 0 \\ \delta \arctan(\rho/2) & \Leftarrow \kappa = 1 \\ \delta \operatorname{artanh}(\rho/2) & \Leftarrow \kappa = -1 \end{cases}$$

$$\delta\rho^2 + \lambda^2(\delta\theta^2 + \sin^2\theta \delta\phi^2) \equiv \frac{\delta\rho^2 + \rho^2(\delta\theta^2 + \sin^2\theta \delta\phi^2)}{1+\kappa\rho^2/4} \equiv \frac{\delta\xi^2 + \delta\eta^2 + \delta\zeta^2}{1+\kappa\rho^2/4}$$

The following versions comprise all possible values of the constant of curvature,  $\kappa$ :

$$\boxed{d\mathbf{T}^2 = dT^2 - \mathcal{E}^2(T)\{\delta\lambda^2/(1-\kappa\lambda^2) + \lambda^2(\delta\theta^2 + \sin^2\theta \delta\phi^2)\}}$$

$$\boxed{d\mathbf{T}^2 = dT^2 - \mathcal{E}^2(T)\{\delta\rho^2 + \rho^2(\delta\theta^2 + \sin^2\theta \delta\phi^2)\}/(1+\kappa\rho^2/4)}$$

$$\boxed{d\mathbf{T}^2 = \mathcal{H}^{-2}(\mathbf{T})[dT^2 - \{\delta\xi^2 + \delta\eta^2 + \delta\zeta^2\}/(1+\kappa\rho^2/4)]}$$

In the  $\mathbf{T}$ -scale, the expansion of cosmos is explained by the shrinking of its atoms!

## 4. The Simple Big Bang Model

In what follows we will throw light on **RWM** by discussing some simple world models. One of the simplest is Milne's uniform expansion model ( $\mathcal{E} \equiv T$ ), adopted by Prokhovnik & al.:<sup>5</sup>

$$\mathcal{E} \equiv T \Rightarrow : R = T\rho, \quad T/t_o = 1 + \ln(T/t_o), \quad \mathcal{H} \equiv dT/dT = e^{(t_o - T)/t_o}$$

Let us first assume that radar signals are exchanged between a pair of observers,  $P$  &  $Q$ , in (1x1) *time-space*. Suppose that a photon  $\phi$  is emitted from  $P$  at  $T_1^p$ , reflected by  $Q$  at  $T = T_2^q$ , and received again by  $P$  at  $T_3^p$ . Then, according to the relativity principle as interpreted by Milne,  $T_3^p$  is the same function of  $T_2^q$  as  $T_2^q$  is of  $T_1^p$  - call it  $S(T) \equiv Te^\sigma$ . Generalizing, and introducing Einsteinian standard coordinates  $t$  &  $r$  for  $P$  (priming those of  $Q$ ), we immediately obtain:

$$\begin{aligned} T_3 &= Te^\sigma \equiv t + r \\ T_1 &= Te^{-\sigma} \equiv t - r \end{aligned}$$

$$t = T \cosh \sigma, \quad r = T \sinh \sigma$$

Let us next assume that  $\sigma$  is not a constant, but a variable; then, by differentiation:

$$\begin{aligned} \delta t &= dT \cosh \sigma + T \delta \sigma \sinh \sigma \\ \delta r &= dT \sinh \sigma + T \delta \sigma \cosh \sigma \end{aligned}$$

$$d\mathcal{T}^2 \equiv \delta t^2 - \delta r^2 = dT^2 - T^2 \delta \sigma^2$$

This invariant may easily be expanded into a full *time-space* of (1x3) dimensions if we put:

$$\delta \sigma^2 \equiv \delta \rho^2 + \sinh^2 \rho (\delta \theta^2 + \sin^2 \theta \delta \phi^2) \equiv \{\delta \xi^2 + \delta \eta^2 + \delta \zeta^2\} / (1 - \rho^2/4)$$

The standard invariant of **SR** may then be transformed into the hyperbolic metric of an expanding universe with expansion function  $\mathcal{E}(T) \equiv T$ , which may be transformed into another hyperbolic metric - viz. that of a stationary universe whose atoms all contract in accordance with the corresponding Hubble function  $\mathcal{H}(T) \equiv e^{(t_o - T)/t_o}$ , where  $t_o$  is a constant of calibration:

$$\boxed{d\mathcal{T}^2 = \delta t^2 - \delta r^2 - r^2(\delta \theta^2 + \sin^2 \theta \delta \phi^2)}$$

$$\boxed{d\mathcal{T}^2 = dT^2 - T^2\{\delta \rho^2 + \sinh^2 \rho (\delta \theta^2 + \sin^2 \theta \delta \phi^2)\}}$$

$$\boxed{d\mathcal{T}_{t_o \equiv 1}^2 \equiv e^{2(T-1)}[dT^2 - \{\delta \xi^2 + \delta \eta^2 + \delta \zeta^2\} / (1 - \rho^2/4)]}$$

The 1st of these metrics, which incorporates the universal constancy of the speed of light, yields an infinity of *private time-spaces*, comprising the flat 3-spaces of all fundamental observers. The second and third metrics both yield a *public time-space*, each containing a curved 3-space: that of the 2nd metric being associated with *T-time*, relative to which atoms keep a constant size while the distances between fundamental observers steadily expand in proportion to  $\mathcal{E} \equiv T$ , with the consequence that *light is stretched*, as suggested by Prokhovnik - and that of the 3rd metric being associated with *T-time*, relative to which distances between fundamental observers remain invariant whereas the sizes of their atomic constituents are steadily contracting in proportion to  $\mathcal{H}_{t_o=1} \equiv e^{1-T}$  in consequence of a *secular reduction* of the velocity of light, as explained by Whitrow.

## 5. The Original Steady State Model

Passing from Milne's world model to that of Gold & Bondi, and of Hoyle, the expansion function  $\mathcal{E}(T)$  is changed from  $T$  to  $e^T$ , to which all steady state models must approximate. So:

$$\mathcal{E}(T) \equiv e^T \equiv dT/dT \equiv \frac{1}{1-T} \equiv \mathcal{H}^{-1}(T)$$

Now  $R \equiv e^T \rho \equiv \tanh r$  is plausible as the proper distance between fundamental particles, just as  $e^{t-T} \equiv 1/\sqrt{1-R^2}$  is a plausible relationship between frame time  $t$  and proper time  $T$ . Therefore, if Bondi & Gold, and Hoyle, want to retain  $dT^2 - e^{2T} \delta\rho^2$  as a fundamental invariant of their model, in face of the definitions  $e^{t-T} \equiv \cosh r$  &  $e^T \rho \equiv \tanh r$ , they must also accept:

$$dT = \delta t - \tanh r \delta r \quad \Downarrow \quad e^T \delta\rho = \delta r - \tanh r \delta t$$

$$d\mathcal{T}^2 \equiv \frac{\delta t^2 - \delta r^2}{\cosh^2 r} = \underline{dT^2 - e^{2T} \delta\rho^2} = \frac{\delta T^2 - \delta\rho^2}{(1-T)^2}$$

Generalizing to (1x3) dimensional time-space we then obtain the following three metrics, of which the first metric is closest to represent the private 3-spaces of the standard frames of **SR**, whereas the second comprises the public flat 3-space of a universe expanding with  $\mathcal{E}(T) = e^T$  and the third encompasses the public flat 3-space of atoms shrinking in step with  $\mathcal{H}(T) = 1-T$ :

$$\boxed{d\mathcal{T}^2 = [\delta t^2 - \{\delta r^2 + \sinh^2 r (\delta\theta^2 + \sin^2\theta \delta\phi^2)\}]/\cosh^2 r}$$

$$\boxed{d\mathcal{T}^2 = dT^2 - e^{2T} \{\delta\rho^2 + \rho^2 (\delta\theta^2 + \sin^2\theta \delta\phi^2)\}}$$

$$\boxed{d\mathcal{T}^2 = [\delta T^2 - \{\delta\rho^2 + \rho^2 (\delta\theta^2 + \sin^2\theta \delta\phi^2)\}]/(1-T)^2}$$

Here  $d\mathcal{T}^2 = [\delta t^2 - \{\delta r^2 + \sinh^2 r (\delta\theta^2 + \sin^2\theta \delta\phi^2)\}]/\cosh^2 r$  should be compared to the standard invariant of **SR** which is the much simpler one  $d\mathcal{T}^2 = \delta t^2 - \delta r^2 - r^2 (\delta\theta^2 + \sin^2\theta \delta\phi^2)$ . One problem is that standard **SR** presupposes a Euclidean 3-space, and not a Lobachevskian one. More serious seems the problem caused by the external factor  $\cosh^{-2} r$  which is much less akin to the **LT** of **SR** than to the Voigt transformations of some unknown underlying ether theory.

May be, if evidence accumulates, we shall have to recur once more to the ether hypothesis. However, since neither **LT** nor **TT** have yet been finally disproved - and since the "steady state" assumption does not necessarily exclude the common conjecture that the universe is of finite age and originated in a "big bang" - it is worth while to search for an alternative "steady state" model that is not at variance with  $d\mathcal{T}^2 = \delta t^2 - \delta r^2$ . As a step on the way I shall propose the model:

$$\mathcal{E}(T) \equiv e^T \equiv \frac{1}{1-T} \equiv \mathcal{H}^{-1}(T)$$

$$\boxed{d\mathcal{T}^2 = \delta t^2 - \{\delta r^2 + \sinh^2 r (\delta\theta^2 + \sin^2\theta \delta\phi^2)\}}$$

$$\boxed{d\mathcal{T}^2 = [dT^2 - e^{2T} \{\delta\rho^2 + \rho^2 (\delta\theta^2 + \sin^2\theta \delta\phi^2)\}]/(1 - e^{2T} \rho^2)}$$

$$\boxed{d\mathcal{T}^2 = [\delta T^2 - \{\delta\rho^2 + \rho^2 (\delta\theta^2 + \sin^2\theta \delta\phi^2)\}]/\{(1-T)^2 - \rho^2\}}$$

This, at the very least, is compatible with the standard invariant  $d\mathcal{T}^2 = \delta t^2 - \delta r^2$  of **SR**. However, if interpreted solely by means of  $e^{t-T} \equiv \cosh r$ ,  $e^T \rho \equiv \tanh r$ , the model is flawed:  $T$  cannot pass for a genuine cosmic time since, in general,  $d\mathcal{T} \neq dT$  for  $\delta\rho \propto \delta\theta \propto \delta\phi \propto 0$ .

## 6. An Alternative Steady State Model

Let us start from scratch by adopting  $d\mathcal{T}^2 = \delta t^2 - \delta r^2$  of **SR**. However, since we need not accept that standard frames are flat, we are free to assume that existing reality is better described, when referring to frame time  $t$ , by taking the geometry of frames, or 3-spaces, to be hyperbolic:

$$\boxed{d\mathcal{T}^2 \equiv \delta t^2 - \{\delta r^2 + \sinh^2 r (\delta\theta^2 + \sin^2\theta \delta\phi^2)\}}$$

Now the shortcoming alluded to in §5 can be remedied by adopting the following definitions:

$$\underline{\rho \equiv \sinh r e^{-t} \equiv \tanh r e^{-t} \equiv 2 \tanh \frac{r}{2} e^{-T} \equiv R e^{-T}}$$

From the above definitions we immediately derive the following relationships:

$$\underline{e^t \delta\rho = \delta r \cosh r - \delta t \sinh r = \delta r \cosh^{-1} r - \delta t \sinh r = \delta r - dT \sinh r}$$

Now, for  $\delta\rho = 0$  (*fundamental observers*),  $v \equiv \delta r/\delta t \equiv \tanh r$ ,  $w \equiv \delta r/dT \equiv \sinh r$ , whence:

$$\delta t/dT = 1/\sqrt{1-v^2} = \sqrt{1+w^2} = \cosh r \equiv \gamma$$

Adopting also  $dT \equiv \gamma \delta\tau$  of Tangherlini & Selleri for  $\delta\rho \neq 0$  (*accidental observers*) we deduce:

$$\underline{dT = \delta t - \delta r \tanh \frac{r}{2} = \gamma \delta\tau}$$

Applying our definitions to the hyperbolic metric above, we derive these further metrics:

$$\boxed{d\mathcal{T}^2 = [\delta t^2 - e^{2t} \{\delta\rho^2 + \rho^2 (\delta\theta^2 + \sin^2\theta \delta\phi^2)\}]/(1 - e^{2t} \rho^2)}$$

$$\boxed{d\mathcal{T}^2 = dT^2 \left\{ \frac{1 + \frac{1}{4} e^{2T} \rho^2 - \frac{1}{2} e^{2T} \rho (\frac{\delta\rho}{dT})}{1 - \frac{1}{4} e^{2T} \rho^2} \right\}^2 - \frac{e^{2T}}{(1 - \frac{1}{4} e^{2T} \rho^2)^2} \{\delta\rho^2 + \rho^2 (\delta\theta^2 + \sin^2\theta \delta\phi^2)\}}$$

$$\boxed{d\mathcal{T}^2 = \frac{\delta T^2}{(1-T)^2} \left\{ \frac{(1-T)^2 + \frac{1}{4} \rho^2 - \frac{1}{2} \rho (\frac{\delta\rho}{dT(1-T)})}{(1-T)^2 - \frac{1}{4} \rho^2} \right\}^2 - \frac{1}{\{(1-T)^2 - \frac{1}{4} \rho^2\}^2} \{\delta\rho^2 + \rho^2 (\delta\theta^2 + \sin^2\theta \delta\phi^2)\}}$$

$$\delta\rho \simeq 0 \Rightarrow \frac{1 + \frac{1}{4} e^{2T} \rho^2 - \frac{1}{2} e^{2T} \rho (\frac{\delta\rho}{dT})}{1 - \frac{1}{4} e^{2T} \rho^2} \simeq 1, \quad \delta\rho_{dR \simeq 0} \simeq -\rho dT \Rightarrow \frac{1 + \frac{1}{4} e^{2T} \rho^2 - \frac{1}{2} e^{2T} \rho (\frac{\delta\rho}{dT})}{1 - \frac{1}{4} e^{2T} \rho^2} \simeq \frac{1 + th^2 \frac{r}{2}}{1 - th^2 \frac{r}{2}} = \gamma$$

So, for fundamental particles:  $d\mathcal{T} = dT$ , whereas, for accidental particles at rest:  $d\mathcal{T} = \gamma dT$ . Please, notice that  $T$  is a genuine cosmic time on account of  $dT \propto d\mathcal{T}$  for  $\rho \propto \theta \propto \phi \propto const$ .

Of our four metrics, the 1st one applies to all observers, whether fundamental or accidental. We derive the 2nd metric from the 1st by means of  $\rho \equiv \sinh r e^{-t} \equiv th r e^{-t}$ , just as we derive the 3rd metric from the 2nd by means of  $\rho \equiv th r e^{-t} \equiv 2 th \frac{r}{2} e^{-T} \Rightarrow e^t/\sqrt{1-e^{2t}\rho^2} = e^T/(1-e^{2T}\rho^2/4)$ . The 4th metric finally follows from the 3rd by applying the definition  $\mathcal{E}(T) \equiv e^T \equiv \frac{1}{1-T} \equiv \mathcal{H}^{-1}(T)$ . In this way the **SR**-like invariant  $d\mathcal{T}^2 = \delta t^2 - \delta r^2 - \sinh^2 r (\delta\theta^2 + \sin^2\theta \delta\phi^2)$  is transformed into two new metrics of negative variable curvature, where one - the 3rd - is representing a universe in exponential expansion, while another - the 4th - representing the same universe as stationary, although with shrinking atoms.

What is particularly interesting about this model is that the definitions used to interpret the basic **SR**-like invariant  $d\mathcal{T}^2 = \delta t^2 - \delta r^2 - \sinh^2 r (\delta\theta^2 + \sin^2\theta \delta\phi^2)$  shows a distinct similarity to the definitions introduced above, §2, to the purpose of re-interpreting the standard **SR** of Einstein & Minkowski. Hence we shall suggest, in line with Selleri (Tangherlini), that the proper **SR**-like transformations to be used between accidental observers who do not belong to the substratum are these ones:

$$\boxed{r' = \gamma(r - v\tau), \quad T' = T = \tau\gamma = \tau'\gamma', \quad r = \gamma'(r' - v'\tau')}$$

It is interesting that three of our metrics above allow the entire infinite universe to be enclosed within the confines of a *geometrical pseudo-sphere* (a perfect *black hole*), thus offering a synthesis between the cosmological views of two famous Greek philosophers, Parmenides & Heraclitus.

## 7. Conclusion

Any world model with an **RWM** type of metric can be described in three different ways:

The **1st** way of description is based on the idea that the speed of light is an universal constant. It is a natural claim that the metric common to all observers, fundamental or accidental, is reducible to:

$$dT^2 = \delta t^2 - \delta r^2 \text{ for } \delta\theta = \delta\phi = 0$$

This, however, is not the case for that of the original steady state model; its metric is not **SR**-like. The safest way to ensure that the claim is fulfilled is, of course, to start with the standard metric.

As pointed out, this timespace is *private* as a consequence of the retardation of clocks and the contraction of rods or - with a single expression - the relativization of our metrical units.

An **SR**-like metric in itself says nothing about cosmic expansion or atomic contraction. In spite of Whitrow's claim to have derived **RWM** from the relativistic  $\gamma$ -factor which is clearly equivalent to an **SR**-like metric, it is hard to see more than a mere analogy between these two:

$$\boxed{dT^2 = \gamma^{-2}\delta t^2 = \delta t^2 - \delta r^2}$$

$$\boxed{dT^2 = \delta T^2 - \mathcal{E}^2(T)\delta\sigma^2 = \mathcal{H}^{-2}(T)\{\delta T^2 - \delta\sigma^2\}}$$

Moreover, if the former applies to accidental and fundamental observers without discrimination, whereas the latter is assumed to represent the structure of the substratum of fundamental observers, we shall obviously need some rules of interpretation which can take us from the former to the latter by explicitly narrowing the perspective, thus giving privilege to fundamental observers. Our presentation of the big bang model of Milne & Prokhovnik, of the steady state model of Bondi & Gold, and of our own alternative to the latter model, offers precisely such rules.

The **2nd** and **3rd** ways of description treat the universe as a spatial totality unfolding in a common world time, thereby invoking the idea of *public* timespace.

According to the **2nd** way, the spatial extension of the substratum is assumed to expand relative to the extension of its material content which is determined by the structure of its atoms, i.e. the distances between fundamental observers are increasing relative to their internal structure. So the proper distance between two fundamental observers is given by  $\mathcal{R} \equiv \mathcal{E}(T)\sigma$ , where  $\mathcal{E}$  is the expansion function, and  $\sigma$  is a fixed coordinate characterizing the "moving" observer.

According to the **3rd** way, the spatial extension of the substratum is taken to be stationary while the dimensions of its contents contract secularly in pace with the reduction of atoms, i.e., the inner structure of fundamental observers is shrinking relative to their proper distances. This shrinking takes place in step with  $\mathcal{H}(T)$ , where  $\mathcal{H}$  is the Hubble function, in fact the inverse of the function  $\mathcal{E}$ , and where T is an auxiliary time scale defined by:  $T \equiv \int dT/\mathcal{E}(T) + const.$

### References

1. F. Selleri, 1997, in: **Found.Phys.Lett.**10,1.
2. Cf. my "Ideas of Cosmology", 2000, in Duffy & Wegener: *Recent Advances in Relativity Theory Vol.I*, Hadronic Press.
3. See ch.V, §6, of (Edinb. 1961), or ch.6, §6.4, of (Oxf. 1980).
4. E.A. Milne, 1948/1951: *Kinematic Relativity*, Oxford Univ.Pr.
5. S.J. Prokhovnik, 1967: *The Logic of Special Relativity*, Cambr.Univ.Pr.
6. K.R. Popper, 1940: "Interpretation of Nebular Red-Shifts", **Nature** 3663.