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AXIOMS FOR TEMPO-SPATIAL COSMOLOGY

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Summary:

In the present paper it is shown how it is possible by means of a time-based concept of equidistance to construct a spatial geometry for relativistic cosmology. In analogy to a sphere defined as the geometrical site for all those points which are equidistant from a given point, we construct a plane as the site for all those points that are equidistant from two points, and a line as the site for all those points equidistant from three points.

Having defined perpendicularity and parallellity, we proceed to define the Cosmic Substrate in a way analogous to the cosmological principle of the Cusan. Assuming that we can always construe the center point for any three non-collinear members of this Substrate, we can prove simultaneity to be universally transitive for all members of the Substrate if the simultaneity is defined indirectly by means of equidistance.

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Oral Preamble

An axiomatization of relativistic cosmology may be construed with various aims in mind. One goal is to codify pet ideas and entrenched theories thus giving boost to scientific dogma. Quite another is to invent a model depicting some basic traits of the universe in order to test that structure against experience. It is the second purpose that has motivated my article.

Instead of repeating what is written in my paper, I prefer to say a little more about why I find it important to put focus on this very general structure displaying these particular traits. Like the philosopher Bergson I always felt that the socalled "spatialization of time" is a great mistake on behalf of science, indeed the ultimate catastrophe. The idea of a "block universe" wherein nothing happens, everything existing of eternity though in a timeless way, is foreign to experience, and the phantasy of "time-warps" and "space-time tunnels" leading to other worlds is, in my opinion, nothing but idle speculation bolstered up with subtle mathematics.

Eddington once said: "In physics everything depends on the insight with which the ideas are handled *before* they reach the mathematical stage". These words are true, wise and pertinent. But, like Eddington, most physicists still turn to Einstein, as if *his* ideas are the highest wisdom. Now, according to Einstein, time is what is read off our clocks and, if our clocks are retarded, time itself must be dilated (this inference was made by Minkowski, then adopted by Einstein). Further Maxwell's equations involve a constant c that is naturally interpreted as a limiting speed, viz., that of light. What is more natural, then, than to insist that time is another form of space? Long forgotten is the doctrine of Descartes, that mind and matter are two different kinds of being having one property in common, viz. their temporal duration or "durée", as Bergson said. Equally eclipsed by oblivion is insight of Kant that, whereas the intuition of space yields the (geometric) form of all external experience, the intuition of time yields the (arithmetic) form not only of internal experience, but of all experience, external as well as internal. It was precisely this insight that inspired Hamilton to invent his calculus of quaternions, so fruitful to physics.

But is it not "a reactionary move" to propose an axiomatics to revitalize those outmoded concepts of absolute simultaneity and universal time so characteristic of classical physics? Well, I have not the slightest doubt that professor Nemeti and his followers will consider it that way. As far as I understand, they want to base their axiomatics on the socalled light-cone geometry. The point to be noticed here is that the opening angle of the cone to its axis is representative for the velocity of light: for the one-way velocity of light, that is. Now the standard version of the special theory of relativity (SR) is distinguished from other competing theories not only by its two basic principles, that of relativity and that of the constancy of the velocity of light, but also by the fact that its definition of simultaneity at a distance is determined purely by convention. This, of course, entails that its definition of the one-way velocity of light is conventional too.

That this is so was shown by John Winnie, and in another way by my friend Peter Øhrstrøm. SR is experimentally equivalent to a host of theories, each with its own one-way light velocity. So, according to SR, whereas the average or two-way velocity of light is a universal constant, the one-way velocity of light - hence the opening angle of its light cones - is wholly arbitrary. This, as far as I can see, puts the geometric light-cone enterprise completely in jeopardy.

To this argument it might be objected that, recalling Reichenbach's ϵ-constant, Einstein's choice of $\epsilon = \frac{1}{2}$ is unique in the sense that it accords with the value obtained by infinitely slow clock transport. It may further be adduced that Malament has presented a famous argument to the effect that the value $\epsilon = \frac{1}{2}$ is the only one that allows for a reversal of the direction of time. Finally, this value appears to be indispensable to the standard definition of a reference frame. However, such objections are nothing but subterfuges. Just like the definition of simultaneity at a distance, the one-way velocity of light for a certain distance is either conventional, or it is not. If we stick to SR it is certainly conventional, that is, indifferent to experiment and observation, and then the standard concept of a reference frame is nothing but a convenient fiction, or a fake. If it can be shown that it is not conventional, we shall have to search for an alternative theory, involving a different concept of reference frame than the usual one of standard relativity.

Now dr. Rowlands, in his paper, suggests that gravitational effects may be instantaneous. This, in my opinion, is a suggestion that might well be true and should not be lightly dismissed. He admits that "there is, therefore, an incompatibility between the space-time structure of our observations (supposing it to be that of standard SR) and the space-time structure we require to set up our gravitational equations" but, adds he, "this is not an uncommon occurrence in physics and there is a well known solution". To a logician such words may sound not a little distressing. It is, indeed, well known that physicists - instead of taking incompatibilities and contradictions for what they are, namely, incompatibilities and contradictions - try by all means to evade them by making distinctions so subtle that no one can sort them out. This applies in particular to the attempts at unifying GR with QT. Very few seem willing to face the fact that GR and QT are incompatible and that all attempts to unify them inevitably lead us into a mess of contradictions. So I agree with dr. Rowlands that "quantum gravity is a meaningless idea". But I think the same holds of his own proposal that "if we apply a Lorentzian (read: Minkowskian, MW) space-time to a system subject to non-local gravitation", this obvious incompatibility can be amended by introducing something called "fictitious effects" to compensate for an unmitigated contradiction. Indeed, it is obvious that in order to defend the view that gravitational effects are instantaneous, the notion of simultaneity at a distance must be uniquely definable, and this it is not within SR. Here standard logic certainly holds sway, and I see no reason at all to endorse current attempts to modify basic logic in accordance with contemporary physics, whether relativistic or quantal. (Please, do not take my criticism personally, Peter - I find your new book *0* $\rightarrow \infty$ fascinating!) So, what can we do to incorporate the idea that gravitational effects are instantaneous?

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Another of my friends, professor Selleri, points to what I think is a much better solution. Having in his forthcoming new book on *Weak Relativity* assembled and discussed a number of observations and experiments in support of the revolutionary idea that simultaneity, after all, is absolute, he proceeds to develop some new inertial transformations to accomodate that idea. Accepting the evidence here accumulated, I agree that we shall have to search for a new theory. An essential characteristic of the inertial transformations proposed and generalized in his book is that the dependence of the temporal coordinate on the (longitudinal) spatial one so distinctive of standard SR is here suspended, so that only the well known γ-factor of SR remains relevant. The same remark applies to another non-standard theory developed by our common friend, Tom Phipps, who in his monumental book entitled *Heretical Verities* has constructed a full-fledged socalled neo-Hertzian electrodynamics in due respect to almost all current relativistic evidence. Regrettably, however, these two heretic spirits do not agree concerning the spatial coordinate. Further, they are both reluctant to accept the full validity of a strong relativity principle.

In my own opinion, this is a serious drawback of their respective non-standard theories. If I had shared their view, I might not have been that eager to invite our guest professor Ungar. In any case I agree with professor Ungar that the standard Lorentz transformations of SR should be applicable to all equivalent observers in a universe which is in a state of uniform expansion. Of course, this latter condition imposes a problematic restriction on our cosmological theories; so we shall have to search for other transformations if our universe, as it seems, is accelerating. but the need for new transformations is also urgent if the observers involved are not equivalent. Now the statement that the universe is accelerating must refer to an accepted standard, and the usual standard is given by the size of our own body, i.e., by the size of its constituting atoms. When this standard is extrapolated ad infinitum in 3-space we get our usual reference frames, and if such frames are synchronized by the usual convention, our choice of an origo seems free. What I surmise is that this is a mistake that entails the dissolution of classical simultaneity; so if, with Selleri and Phipps, we want to keep simultaneity absolute, this mistake should be avoided. I agree that all this sounds very strange. How can I both accept a weak concept of relativity for the reason that it allows simultaneity to be absolute and yet endorse a strong relativity principle? An explanation is obviously urgent. But my point is that a distinction must be made.

A way out of the impasse was shown long ago by E.A. Milne in his *Kinematic Relativity*. In this classical monograph he proposed a world model whose structure is determined by an infinite substratum of perfectly equivalent fundamental particles exposed to uniform dispersion. This substratum, the members of which are subject to the strong relativity principle, was then supposed to be covered by a layer of accidental particles subject to a weaker kind of relativity. Milne's universe thus consists of two sets of particles: the substratum of fundamental particles subject to strong relativity as formulated in the principle of cosmic isotropy, and a further layer of accidental particles breaking the cosmic symmetry, and hence only subject to weak relativity.

Milne denounced the Friedmann equations of GR, so his model is not a Friedmannian one. Instead of assuming gravitation, hiding it in the curvature of space, it was his aim to deduce it. He began by deriving the Lorentz transformations and showing them to hold good between the comoving frames of all fundamental observers, *implying that each frame has its natural origo in the only fundamental observer at rest in that frame,* all others receding with uniform velocity. His assistant Walker then showed the standard coordinates of SR to be transmutable into some other coordinates mapping the universe as being in a state of uniform expansion in cosmic time. As I see it, the difference between the many ***private*** Lorentzian 3-spaces, each Lorentz frame correlated with its own Einsteinian t-time, and the single, unique and ***public*** Robertson-Walker 3-space correlated with the cosmic T-time of Walker, is also the key to an understanding of the ingenious non-commutative non-associative algebra invented and studied by professor Ungar. The Milne model, which is a very clean one, was later adopted by Törnebohm and Prokhovnik; it may be described as having its origin not in a "big bang", but in a transcendent point-event. Applying his cosmological principle to the Lorentz transformations, and making an important distinction between ***world map*** (the universe as it is in itself at an instant of cosmic time) and ***world view*** (the universe as it appears to an observer, sliced up in different temporal layers), cf. the famous one of Kant between "das Ding an sich" and "das Ding für uns", Milne was able to describe precisely how his world model of uniform dispersion relates to observation.

How did this remarkable feat fall into oblivion, so that Milne and his kinematic relativity theory is hardly mentioned in more recent expositions of cosmology? Well, one explanation is that the fame of Einstein has reached mythological dimensions overshadowing that of all other. In our modern culture, what is greater than to become depicted on the side of a shopping bag? Every child knows the name of Einstein, but who cares about that of Plato, except philosophers? Another explanation is that Milne's gravitational theory did not stand up to observational test, but was surpassed and excelled in that respect by Einstein's. However, this is just another myth. You must allow a theory to become developed, and it was reported already by Whittaker in his monumental *History of the Theories of Aether and Electricity, vol.2,* that Walker had suggested a variational principle in conformity with SR which reproduced the observational results of GR, so he issued a much ignored warning: don't accept a theory with only scant experiental support. It later became almost a sport to construe formulae based on SR and imitating the results of GR. I have offered two myself, and the whole issue has been discussed thoroughly by Rowlands. *Here I only want to stress the ingenuity of Milne's basic idea: to derive gravity from asymmetry.* We have recently learned so much about broken symmetry leading to differentiation of forces: strong, weak, electromagnetic, etc., but hitherto noone has succeeded to incorporate gravitation. The project of quantum gravity is still unsolved, despite the endless efforts of mathematicians. But a solution may be close at hand and much simpler than expected: just follow Milne!

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According to Milne's kinematic relativity theory, the substratum is an ideal, infinite and dense, set of equivalent particles subject to hydrodynamic continuity and obeying Hubble's law. Since he assumed the universe to be in a state of *uniform dispersion* **-** expressed with the metric of Walker: in a state of *uniform expansion* **-** this universe embodies a perfect *compass of inertia*, to use a felicitous phrase of Gödel. This means that it is impossible to construe an inertial frame which is not in the end identifiable with the frame comoving with some fundamental observer. But, as already mentioned, only one fundamental observer can be at rest in any inertial frame: this observer constitutes the natural origo of that frame, and we are not free to choose any other. The substratum, we said, is covered by a layer of accidental particles. Now, in contrast to the fundamental particles which are at rest, each one in its own comoving frame, the position and velocity of an accidental particle is completely described by reference to two fundamental ones: viz. that with which is momentarily coincides and that relative to which it is momentarily at rest, and the closer these two are to each other, the more the particle will tend to fundamental status. Milne's model, like any other model of the universe, is subject to the conservation of energy. Now an accidental particle, due to its velocity, possesses a certain amount of kinetic energy as estimated by that fundamental observer with which it momentarily coincides. But fundamental observers are equivalent. The accidental particle therefore possesses exactly the same amount of energy as estimated by any other fundamental observer, and only its form may seem different. How, in particular, must its energy be perceived by that observer relative to which it is at rest? Only one possibility is left open to him: the energy must appear as being potential, or dynamic. So, *what to the first observer appears as* ***inertia*** *to the second must appear as* ***gravity***!

I agree with professor Ungar that there is an intimate connexion between special relativity and hyperbolic geometry. The same does dr. Barrett, and so did Milne. But there is a difference. Whereas professor Ungar ascribes a hyperbolic character to the *velocity space* of SR, dr. Barrett also tends to view its *position space* as being hyperbolic. Milne, by contrast, speaks of a choice. In this case I side with Barrett; but my reasons for that position are probably different from his. My point is that - having earlier investigated the possibility of new steady state world models where the universe either is, or tends to be, in a sort of steady state - *if* we assume that the universe is subject to accelerated dispersion/expansion, and *if* we suppose that this acceleration either already obeys, or at least approximates towards obeying, an exact version of Hubble's law involving a precise proportionality of velocity to distance, *then why* accept that the structure of position space is different from that of velocity space? No, they must be identical.

With Milne, gravitation is explained by inertia, not the other way round. Only accidental particles are exposed to gravitation. Atomic clocks of fundamental observers keep the same rate. Gravity is caused by asymmetry. Cosmic time is conditioned by symmetry.

§1. *INTRODUCTION.*

The Greeks prescribed that proofs of geometry should be made solely by means of a ruler and a pair of compasses. In this paper it will be shown how the ruler can be dispensed with, and how all concepts of geometry can be constructed by means of "a temporal pair of compasses", in a way reminiscent of, but also different from, the method sketched by Georg Mohr [1672].

Geometry treats of spatial structure and, according to Descartes, space is real, because extension is a sign of substance on a par with cogitation. But Leibniz denied this, arguing that extension, being infinitely divisible, cannot be substantial. According to Leibniz, space is not real, but neither is it illusive; rather it is well-founded appearance. To this view he was inspired by Plato who, in his *Timaios*, held that space is neither pure concept (*idéa*), nor pure appearance (*fainómenon*), but "something in between" so that, seeing it "as in a dream", we cannot tell what it is, but feel that it has to be, in order for events to take place. Thus, to Plato, space is necessary in order to give room for events, whereas time alone is "an image in motion of eternity".

Contrary to common prejudice the Platonic view of space is supported by modern science Thus, in their excellent monograph *On General Relativity*, Mercier, Treder & Yourgrau [1979] unanimously insisted that "there is no such thing as real space" (p.134). Further, Mercier [2000], in a more philosophical article, advocated the view that 'space-time' should be reconstructed as 'time-space', or 'super-time'. But, more than half a century before this, Milne already suggested the same idea, first in several papers, partly written in collaboration with his student Whitrow, then in his own ingenious *Kinematic Relativity* [1948/1951], wherein he constructed an entirely new cosmology from first terms based on time as its fundamental concept. This cosmology was described by Merleau Ponty [1965] as a "Leibnizian Monadology" translated into mathematics. The present paper may be read as a modest attempt to elaborate on the very same idea.

According to Whitrow [1972], "Leibniz's principle (of the pre-established harmony, MW) is equivalent to the postulate of a single universal time; we must therefore discard this principle if we are to reconcile Leibniz's way of regarding time with Einstein's theory of relativity" - cf. also Wegener [1993]. But why try to reconcile Leibniz with Einstein? As shown by Walker, another student of Milne's, who (independently of Robertson) invented the relativistic standard metric of universes which are everywhere isotropic, such universes not only allow, but outright demand, an universal time parameter to determine the spatial scale factor. And, as demonstrated by Törnebohm [1963], even special relativity gives place to two distinct concepts of time and two different definitions of simultaneity: one relative, another absolute. So, as argued by myself, Wegener [2004], we have every reason to search for a new formalization of relativity consistent with the idea of a Cosmic Time - even if this goes against the spirit of Einstein.

§2. FORMAL PRESENTATION

In what follows I have found it more important to convey to an impression of the main ideas rather than to offer an impeccable presentation of some full-fledged formalization.

The axiomatics is sketched by means of 1st order predicate logic (FOL) or quantification theory (QT) supplemented with basic set theory, $\forall$ being the universal quantor, $\exists$ the existential quantor, and $\Rightarrow, \wedge, \vee, \neg, \Leftrightarrow$ being the constants of implication, conjunction, disjunction, negation, and equivalence. $\forall x : Ax \Rightarrow Bx$, $\exists x : Ax \wedge Bx$ are well-formed formulae, and $\{., x, y, z, .\}$, $\{x|(\textit{description})\}$ are simple sets. Pauses are marked out by points. Quantors are omitted in the definitions. Definitions are indicated by $. \equiv .$ which - depending on the context - may be read either as $=_{df}$ (i.e. identity: "**is**/**are**") or as $\Leftrightarrow_{df}$ (i.e. equivalence: "**iff**").

Df.1: *The Universe.*

$$\boldsymbol{Universe}\ . \equiv . \left\{ \mathrm{P}^i \,|\, \mathrm{P}^i \equiv \{.., P^i_1, P^i_2, P^i_3, ..\} \right\}$$

Translation: The universe **is** the set of all existing points, or particles - or particle-observers, or observer-particles, or monads - which themselves **are** (consist of) sets of events of observation.

AX.1 *Communication.*

$$\forall \mathrm{P}, \forall \mathrm{P}', \forall P_a :\ P_a \in \mathrm{P}\ . \Rightarrow .\ \exists 1\, P'_e :\ P'_e \in \mathrm{P}' \wedge P'_e \rightarrow P_a$$

Translation: For any pair of particles, P & P′, and for any event P_a in P (absorption of a signal), there was one, and only one, event P'_e in P′ (emission of a signal) which was the cause of P_a.
Comment: One should distinguish between *backwards causality*, which is the inference from an effect to its cause, and *forwards causality*, which is the inference from a cause to its effect; here backwards causality is preferable, since forwards causality would involve us with probabilities. Ax.1 states that any event, e.g. state P_a of observer P, is the causal outcome of preceding states of other particles. Thus all particles in the universe communicate by means of signals.

Df.2 *Causal Precedence.*

$$P_e \preceq P'_a\ . \equiv .\ \exists \mathrm{P}, \exists \mathrm{P}' : P_e \in \mathrm{P} \wedge P'_a \in \mathrm{P}' \wedge P_e \rightarrow \ldots \rightarrow P'_a$$

Translation: For any pair of events, P_e in P and P'_a in P′, we shall say that the event of emission P_e was caused by the event of absorption P'_a, **iff** a signal was emitted from P at event P_e and, possibly after transmission by other events, received by P′ at the event P'_a.

AX.2 *Transitivity of Causal Precedence.*

$$P'_1 \preceq P''_2 \wedge P''_2 \preceq P'''_3\ . \Rightarrow .\ P'_1 \preceq P'''_3$$

Translation: If P'_1 caused P''_2 and P''_2 caused P'''_3 then P'_1 caused P'''_3.

Df.3 *Momentary Coincidence.*

$\mathrm{PP}'(P'_r) = 0 \,.\equiv.\, \exists P_e, \exists P_a : P_e, P_a \in \mathrm{P} \wedge P_e \preceq P'_r \preceq P_a \wedge P_e = P_a$

Translation: The distance between P and P$'$ at event P'_r in P$'$ is zero **iff** there are events P_e and P_a in P, so that P_e caused P'_r which caused P_a, but P_e was indistinguishable from P_a.

Df.4 *Identity of Particles.*

$\mathrm{P} = \mathrm{P}' \,.\equiv.\, \forall P'_r : \mathrm{PP}'(P'_r) = 0$

Translation: Two particles, P & P$'$, are identical **iff** their instantaneous distance is always zero.

Th.1 *Difference of Particles.*

$\mathrm{P} \neq \mathrm{P}' \,.\equiv.\, \exists P'_r : \mathrm{PP}'(P'_r) \neq 0$

Translation: Two particles, P & P$'$, are different **iff** their distance at some event is not zero.
Proof: Follows immediately from df.4 by negation, according to QT.

Df.5 *Momentary Equidistance.*

$\mathrm{PP}'(P'_r) = \mathrm{PP}''(P''_r) \,.\equiv.\, \exists P_e, \exists P_a : P_e, P_a \in \mathrm{P} \wedge P_e \to P'_r \to P_a \wedge P_e \to P''_r \to P_a$

Translation: The distance from P to P$'$ at the event of reflection P'_r in P$'$ equaled the distance from P to P$''$ at the event of reflection P''_r in P$''$ **iff** both events of reflection were caused by the same event of emission P_e in P and coincidentially caused the same event of absorption P_a in P.
Comment: This temporal definition of equidistance is basic to our axiomatization of geometry. Being comparable to the span of "a pair of compasses", this justifies our use of that metaphor. The notion of equidistance entails that all signals are transmitted with equal "two-way speed". We shall later make use of the same notion to define the simultaneity of events at a distance.

Df.6 *Permanent Equidistance.*

$\mathrm{PP}' = \mathrm{PP}'' \,.\equiv.\, \forall P'_r, \forall P''_r : \mathrm{PP}'(P'_r) = \mathrm{PP}''(P''_r)$

Translation: The distances from P to P$'$ & P" remain equal **iff** they are equal at all events.

Df.7 *Momentarily Smaller/Greater Distance.*

$$\mathrm{PP}'(P'_r) < \mathrm{PP}''(P''_r) \,.\equiv.$$
$$\exists P_e, \exists P_a, \exists P_{a'} : P_e, P_a, P_{a'} \in \mathrm{P} \wedge P'_r \in \mathrm{P}' \wedge P''_r \in \mathrm{P}'' \wedge P_e \to P'_r \to P_a \wedge P_e \to P''_r \to P_{a'}$$
$$\wedge \exists \mathrm{P}''', \exists P'''_r : P'''_r \in \mathrm{P}''' \wedge \mathrm{PP}'''(P'''_r) \neq 0 \wedge P_a \to P'''_r \to P_{a'}$$

Translation: Distance PP$'$ at P'_r in P$'$ was smaller than distance PP$''$ at event P''_r in P$''$ or corresp. $\mathrm{PP}''(P''_r)$ was greater than $\mathrm{PP}'(P'_r)$, **iff** both reflection events P'_r & P''_r were caused by the same event of emission P_e in P, but caused successive events of absorption P_a & $P_{a'}$ in P, so that a signal emitted at P_a would arrive at $P_{a'}$ if reflected at P'''_r by a particle P$'''$ at distance PP$'''$(P'''_r).

Df.8 *Permamently Smaller/Greater Distance.*

$\mathrm{PP}' < \mathrm{PP}'' \,.\equiv.\, \forall P'_r, P''_r : \mathrm{PP}'(P'_r) < \mathrm{PP}''(P''_r)$

Translation: PP' remains smaller than PP'' **iff** it is smaller at all events of reflection.

Df.9 *Indirect Momentary Distance.*

$\mathrm{PP}'\mathrm{P}''(P''_r) \,.\equiv.$

$$\exists P_e, \exists P_a, \exists P'_t, \exists P'_{t'} : P_e, P_a \in \mathrm{P} \wedge P'_t, P'_{t'} \in \mathrm{P}' \wedge P''_r \in \mathrm{P}'' \wedge P_e \to P'_t \to P''_r \to P'_{t'} \to P_a$$

Translation: The indirect distance from P *via* P' to P'' at P''_r **is** that measured by a signal sent from emission at P_e in P *via* P'_t in P' to P''_r in P'' and back *via* $P'_{t'}$ in P' to absorption at P_a in P.
Comment: This definition can easily be extended to cover any number of transmitting particles.

Df.10 *Momentary Triangularity.*

$\mathrm{PP}''(P''_r) \leq \mathrm{PP}'\mathrm{P}''(P''_{r'}) \,.\equiv.$

$$\exists P_e, \exists P_a, \exists P_{a'}, \exists P'_t, \exists P'_{t'} : P_e, P_a, P_{a'} \in \mathrm{P} \wedge P'_t, P'_{t'} \in \mathrm{P}' \wedge P''_r, P''_{r'} \in \mathrm{P}'' \wedge$$
$$P_e \to P''_r \to P_a \wedge P_e \to P'_t \to P''_{r'} \to P'_{t'} \to P_{a'} \wedge P_a \preceq P_{a'}$$

Translation: The direct distance from P to P'' at the event P''_r in P'' was less than, or equal to, the indirect distance from P *via* P' to P'' at the event $P''_{r'}$ in P'' **iff** P''_r & $P''_{r'}$ were both caused by the same event of emission P_e in P, the first directly and the other indirectly, *via* the first event of transmission P'_t in P', and P''_r & $P''_{r'}$, then each in its turn caused the events of absorption P_a & $P_{a'}$ in P, the first directly and the other indirectly, *via* the second event of transmission $P'_{t'}$ in P', and the event P_a in P then occurred either before than, or coincided with, the event $P_{a'}$ in P.

Df.11 *Permanent Triangularity.*

$\mathrm{PP}'' \leq \mathrm{PP}'\mathrm{P}'' \,.\equiv.\, \forall P''_r, \forall P''_{r'} : \mathrm{PP}''(P''_r) \leq \mathrm{PP}'\mathrm{P}''(P''_{r'})$

Translation: The direct distance from P to P'' remains less than, or equal to, the indirect distance from P *via* P' to P'' **iff** it is less than or equal to it at any pair of reflection events P''_r & $P''_{r'}$.

Df.12 *Momentary Optical Lines.*

$\boldsymbol{opt}\big(\mathrm{PP}'\mathrm{P}''(P''_r)\big) \,.\equiv.\, \mathrm{PP}''(P''_r) = \mathrm{PP}'\mathrm{P}''(P''_r) \,.\equiv.\, \mathrm{PP}''(P''_r) \leq \mathrm{PP}'\mathrm{P}''(P''_{r'}) \wedge P''_r = P''_{r'}$

Translation: The particles $\mathrm{P}, \mathrm{P}', \mathrm{P}''$ formed an optical line in this order at the event P''_r in P'' **iff** the indirect signal from P to P'' *via* P' returned back to P simultaneously with the direct signal from P to P'', that is, **iff** the relation of momentary triangularity held for $\mathrm{P}, \mathrm{P}', \mathrm{P}''$ at $P''_r = P''_{r'}$.

Df.13 *Permanent Optical Lines.*

$\boldsymbol{opt}(\mathrm{PP}'\mathrm{P}'') \,.\equiv.\, \mathrm{PP}'' = \mathrm{PP}'\mathrm{P}''$

Translation: $\mathrm{P}, \mathrm{P}', \mathrm{P}''$ always form an optical line in this order **iff** they do so at any event P''_r.

Df.14 *Spheres.*

$$\boldsymbol{S}(\mathrm{P},\mathrm{P}') \equiv \{\mathrm{P}^i \,|\, \mathrm{P} \neq \mathrm{P}' \Rightarrow \mathrm{PP}^i = \mathrm{PP}'\}$$

Translation: A sphere (spherical surface) **is** the set (or geometrical site) of all those points that keep the same distance to a given fix-point P as does another given fix-point P' different from P.

Th.2 *Non-identity of Spheres.*

$$\boldsymbol{S}(\mathrm{P},\mathrm{P}') \neq \boldsymbol{S}(\mathrm{P}',\mathrm{P})$$

Translation: A sphere about P with radius PP' differs from a sphere about P' with radius $\mathrm{P}'\mathrm{P}$.

Proof: $\forall\,\mathrm{P}, \forall\,\mathrm{P}' : \mathrm{P} \neq \mathrm{P}' \Rightarrow .\, \boldsymbol{S}(\mathrm{P},\mathrm{P}') = \{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{PP}'\} \neq \{\mathrm{P}^i \,|\, \mathrm{P}'\mathrm{P}^i = \mathrm{P}'\mathrm{P}\} = \boldsymbol{S}(\mathrm{P}',\mathrm{P})$

Df.15 *Similarity of Spheres.*

$$\boldsymbol{S}(\mathrm{P},\mathrm{P}') \simeq \boldsymbol{S}(\mathrm{P}'',\mathrm{P}''') \,.\equiv.\, \forall\,\mathrm{P}^i, \forall\,\mathrm{P}^j : \mathrm{PP}' = \mathrm{PP}^i = \mathrm{P}''\mathrm{P}''' = \mathrm{P}''\mathrm{P}^j$$

Translation: Spheres **are** similar if their defining points are equidistant (their radii are equal).

Cor.1 *Similarity of Spheres is Reciprocal.*

$$\boldsymbol{S}(\mathrm{P},\mathrm{P}') \simeq \boldsymbol{S}(\mathrm{P}',\mathrm{P})$$

Translation: The similarity of different spheres with equal radii is reciprocal.

Proof: Follows immediately from the preceding definition.

Cor.2 *Similarity of Spheres is Transitive.*

$$\boldsymbol{S}(\mathrm{P},\mathrm{P}') \simeq \boldsymbol{S}(\mathrm{P}',\mathrm{P}'') \wedge \boldsymbol{S}(\mathrm{P}',\mathrm{P}'') \simeq \boldsymbol{S}(\mathrm{P}'',\mathrm{P}''') \,.\Rightarrow.\, \boldsymbol{S}(\mathrm{P},\mathrm{P}') \simeq \boldsymbol{S}(\mathrm{P}'',\mathrm{P}''')$$

Translation: The similarity of different spheres with equal radii is transitive.

Proof: Follows immediately from the preceding definition.

Df.16 *Circles.*

$$\boldsymbol{C}(\mathrm{P},\mathrm{P}') \equiv \{\boldsymbol{S}(\mathrm{P},\mathrm{P}') \cap \boldsymbol{S}(\mathrm{P}',\mathrm{P})\}$$

Translation: A circle **is** definable as the intersection between two similar spheres.

Th.3 *Circles as Sets.*

$$\boldsymbol{C}(\mathrm{P},\mathrm{P}') = \{\mathrm{P}^i \,|\, \mathrm{P} \neq \mathrm{P}' \neq \mathrm{P}^i \Rightarrow \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{PP}'\}$$

Translation: A circle as just defined is the set of all points keeping the distance PP' to P & P'.

Proof: $\forall\,\mathrm{P}, \forall\,\mathrm{P}', \forall\,\mathrm{P}'' : \mathrm{P} \neq \mathrm{P}' \neq \mathrm{P}^i \Rightarrow \{\boldsymbol{S}(\mathrm{P},\mathrm{P}') \cap \boldsymbol{S}(\mathrm{P}',\mathrm{P})\} = \big\{\{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{PP}'\} \cap$
$\{\mathrm{P}^i \,|\, \mathrm{P}'\mathrm{P}^i = \mathrm{P}'\mathrm{P}\}\big\} = \{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{PP}' \wedge \mathrm{P}'\mathrm{P}^i = \mathrm{PP}'\} = \{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{PP}'\}$

Df.17 *Planes.*

$$\boldsymbol{Pl}(\mathrm{P},\mathrm{P}') \equiv \{\mathrm{P}^i \,|\, \mathrm{P} \neq \mathrm{P}' \Rightarrow \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i\}$$

Translation: A plane **is** the set of all points which are equidistant from two given points, $\mathrm{P} \neq \mathrm{P}'$.
Comment: The defining points do not belong to the plane defined, i.e.: $\mathrm{P}, \mathrm{P}' \notin \boldsymbol{Pl}(\mathrm{P}, \mathrm{P}')$

Cor.3 *Planes.*
$\boldsymbol{Pl}(\mathrm{P}, \mathrm{P}') \Leftrightarrow \{\mathrm{P}^i \,|\, \mathrm{P} \neq \mathrm{P}' \Rightarrow . \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{PP}' \vee \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i \neq \mathrm{PP}'\}$
Translation: A plane is the set of all points which are equidistant from two given points, $\mathrm{P} \neq \mathrm{P}'$.

Th.4 *Identity of Planes.*
$\boldsymbol{Pl}(\mathrm{P}, \mathrm{P}') = \boldsymbol{Pl}(\mathrm{P}', \mathrm{P})$
Translation: The plane between P and P′ is identical to the plane between P′ and P.
Proof: $\forall \mathrm{P}, \forall \mathrm{P}' : \mathrm{P} \neq \mathrm{P}' \Rightarrow . \{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i\} = \{\mathrm{P}^i \,|\, \mathrm{P}'\mathrm{P}^i = \mathrm{PP}^i\}$

Th.5 *Circles as Subsets of Planes.*
$\boldsymbol{C}(\mathrm{P}, \mathrm{P}') \subset \boldsymbol{Pl}(\mathrm{P}, \mathrm{P}')$
Translation: The circle defined by P, P' is a subset of the plane defined by P, P'.
Proof:
$\boldsymbol{C}(\mathrm{P}, \mathrm{P}') . = . \{\mathrm{P}^i \,|\, \mathrm{P} \neq \mathrm{P}' \Rightarrow \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{PP}'\}$ cf. Th.3
$\boldsymbol{Pl}(\mathrm{P}, \mathrm{P}') \equiv \{\mathrm{P}^i \,|\, \mathrm{P} \neq \mathrm{P}' \Rightarrow . \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{PP}' \vee \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i \neq \mathrm{PP}'\}$ cf. Cor.2
$\forall \mathrm{P}^i : \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{PP}' . \Rightarrow . \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{PP}' \vee \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i \neq \mathrm{PP}'$ QT

Th.6 *Circles as Intersections of Spheres and Planes.*
$\boldsymbol{C}(\mathrm{P}, \mathrm{P}') . = . \boldsymbol{S}(\mathrm{P}, \mathrm{P}') \cap \boldsymbol{Pl}(\mathrm{P}, \mathrm{P}')$
Translation: The sphere $\boldsymbol{S}(\mathrm{P}, \mathrm{P}')$ and the plane $\boldsymbol{Pl}(\mathrm{P}, \mathrm{P}')$ intersects in the circle $\boldsymbol{C}(\mathrm{P}, \mathrm{P}')$.
Proof: $\forall \mathrm{P}, \forall \mathrm{P}' : \mathrm{P} \neq \mathrm{P}' \Rightarrow . \big\{\{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{PP}'\} \cap \{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i\}\big\} = \{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{PP}'\}$

Df.18 *Lines.*
$\boldsymbol{Lin}(\mathrm{P}, \mathrm{P}', \mathrm{P}'') \equiv \{\mathrm{P}^i \,|\, \mathrm{PP}'' < \mathrm{PP}'\mathrm{P}'' \wedge \mathrm{P}'\mathrm{P} < \mathrm{P}'\mathrm{P}''\mathrm{P} \wedge \mathrm{P}'\mathrm{P}'' < \mathrm{P}'\mathrm{PP}'' . \Rightarrow \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{P}''\mathrm{P}^i\}$
Translation: A line **is** the set of all points equidistant from three given points not in optical line.
Comment: The defining points do not belong to the line defined, i.e.: $\mathrm{P}, \mathrm{P}', \mathrm{P}'' \notin \boldsymbol{Lin}(\mathrm{P}, \mathrm{P}', \mathrm{P}'')$

Th.7 *Identity of Lines.*
$\boldsymbol{Lin}(\mathrm{P}, \mathrm{P}', \mathrm{P}'') = \boldsymbol{Lin}(\mathrm{P}', \mathrm{P}'', \mathrm{P}) = \boldsymbol{Lin}(\mathrm{P}'', \mathrm{P}, \mathrm{P}') =$
$= \boldsymbol{Lin}(\mathrm{P}'', \mathrm{P}', \mathrm{P}) = \boldsymbol{Lin}(\mathrm{P}', \mathrm{P}, \mathrm{P}'') = \boldsymbol{Lin}(\mathrm{P}, \mathrm{P}'', \mathrm{P}')$
Translation: The line defined by $\mathrm{P}, \mathrm{P}', \mathrm{P}''$ is identical to that defined by $\mathrm{P}', \mathrm{P}'', \mathrm{P}$ which is identical to the one defined by $\mathrm{P}'', \mathrm{P}, \mathrm{P}'$, i.e., a line is indifferent to the order of its defining points.
Proof: $\{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{P}''\mathrm{P}^i\} = \{\mathrm{P}^i \,|\, \mathrm{P}'\mathrm{P}^i = \mathrm{P}''\mathrm{P}^i = \mathrm{PP}^i\} = \{\mathrm{P}^i \,|\, \mathrm{P}''\mathrm{P}^i = \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i\} =$
$= \{\mathrm{P}^i \,|\, \mathrm{P}''\mathrm{P}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{PP}^i\} = \{\mathrm{P}^i \,|\, \mathrm{P}'\mathrm{P}^i = \mathrm{PP}^i = \mathrm{P}''\mathrm{P}^i\} = \{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}''\mathrm{P}^i = \mathrm{P}'\mathrm{P}^i\}$

Th.8 *Intersecting Planes.*

$\boldsymbol{Lin}(\mathrm{P},\mathrm{P}',\mathrm{P}'') = \{\boldsymbol{Pl}(\mathrm{P},\mathrm{P}') \cap \boldsymbol{Pl}(\mathrm{P},\mathrm{P}'')\}$

Translation: The line defined by equidistance to $\mathrm{P},\mathrm{P}',\mathrm{P}''$ is identical to the intersection of the plane defined by equidistance to P,P' and the plane defined by equidistance to P,P''.

Proof: $\forall\,\mathrm{P},\mathrm{P}' : \mathrm{P} \neq \mathrm{P}' \Rightarrow . \{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{P}''\mathrm{P}^i\} = \big\{\{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i\} \cap \{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}''\mathrm{P}^i\}\big\}$

Th.9 *Lines as Subsets of Planes.*

$\boldsymbol{Lin}(\mathrm{P},\mathrm{P}',\mathrm{P}'') \subset \boldsymbol{Pl}(\mathrm{P},\mathrm{P}')$

Translation: The line defined by $\mathrm{P},\mathrm{P}',\mathrm{P}''$ is a subset of the plane defined by P,P'.

Proof: $\boldsymbol{Lin}(\mathrm{P},\mathrm{P}',\mathrm{P}'') = \{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{P}''\mathrm{P}^i\} \subset \{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i\} = \boldsymbol{Pl}(\mathrm{P},\mathrm{P}')$

Th.10 *Planes as Sets of Lines.*

$\forall\,\mathrm{P},\forall\,\mathrm{P}',\forall\,\mathrm{P}'' : \mathrm{PP}'' = \mathrm{P}'\mathrm{P}'' . \Rightarrow . \boldsymbol{Pl}(\mathrm{P},\mathrm{P}') = \{\mathrm{P}''|\,\boldsymbol{Lin}(\mathrm{P},\mathrm{P}',\mathrm{P}'')\}$

Translation: Granted $\mathrm{PP}'' = \mathrm{P}'\mathrm{P}''$, $\boldsymbol{Pl}(\mathrm{P},\mathrm{P}')$ is the set of lines $\{\mathrm{P}''|\,\boldsymbol{Lin}(\mathrm{P},\mathrm{P}',\mathrm{P}'')\}$.

Proof: $\forall\,\mathrm{P},\forall\,\mathrm{P}',\forall\,\mathrm{P}'' : \mathrm{P} \neq \mathrm{P}' \neq \mathrm{P}''. \Rightarrow\ :\ \mathrm{PP}'' = \mathrm{P}'\mathrm{P}'' . \Rightarrow . \boldsymbol{Pl}(\mathrm{P},\mathrm{P}') =$
$\{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i\} = \{\mathrm{P}'' \,|\, \{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{P}''\mathrm{P}^i\}\} = \{\mathrm{P}''|\,\boldsymbol{Lin}(\mathrm{P},\mathrm{P}',\mathrm{P}'')\}$

AX.3 *Lines and Optical Lines.*

$\forall\,\mathrm{P}^1,\forall\,\mathrm{P}^2,\forall\,\mathrm{P}^3 :\ \exists\,\mathrm{P},\exists\,\mathrm{P}',\exists\,\mathrm{P}'' : \mathrm{P}^1,\mathrm{P}^2,\mathrm{P}^3 \in \boldsymbol{Lin}(\mathrm{P},\mathrm{P}',\mathrm{P}'') . \Leftrightarrow .$
$\exists\,\boldsymbol{opt}(\mathrm{P}^1\mathrm{P}^2\mathrm{P}^3) \vee \exists\,\boldsymbol{opt}(\mathrm{P}^3\mathrm{P}^2\mathrm{P}^1) \vee \exists\,\boldsymbol{opt}(\mathrm{P}^2\mathrm{P}^3\mathrm{P}^1) \vee$
$\exists\,\boldsymbol{opt}(\mathrm{P}^1\mathrm{P}^3\mathrm{P}^2) \vee \exists\,\boldsymbol{opt}(\mathrm{P}^3\mathrm{P}^1\mathrm{P}^2) \vee \exists\,\boldsymbol{opt}(\mathrm{P}^2\mathrm{P}^1\mathrm{P}^3)$

Translation: It holds for any triplet $\mathrm{P}^1,\mathrm{P}^2,\mathrm{P}^3$ that **iff** it belongs to the line defined by some other triplet $\mathrm{P},\mathrm{P}',\mathrm{P}''$, then the first triplet constitutes an optical line either in the order $\mathrm{P}^1\mathrm{P}^2\mathrm{P}^3$ or its opposite, or in the order $\mathrm{P}^2\mathrm{P}^3\mathrm{P}^1$ or its opposite, or in the order $\mathrm{P}^3\mathrm{P}^1\mathrm{P}^2$ or its opposite.

Df.19 *Perpendicularity.*

$\boldsymbol{Pl}(\mathrm{P}^1,\mathrm{P}^2) \perp \boldsymbol{Lin}(\mathrm{P},\mathrm{P}',\mathrm{P}'') . \equiv . \mathrm{P}^1,\mathrm{P}^2 \in \boldsymbol{Lin}(\mathrm{P},\mathrm{P}',\mathrm{P}'') \vee \mathrm{P},\mathrm{P}',\mathrm{P}'' \in \boldsymbol{Pl}(\mathrm{P}^1,\mathrm{P}^2)$

Translation: The plane defined by $\mathrm{P}^1,\mathrm{P}^2$ is said to be perpendicular to the line defined by $\mathrm{P},\mathrm{P}',\mathrm{P}''$ (not collinear) **iff** either: $a)$ the points $\mathrm{P}^1,\mathrm{P}^2$ belong to the line defined by $\mathrm{P},\mathrm{P}',\mathrm{P}''$, or: $b)$ the points $\mathrm{P},\mathrm{P}',\mathrm{P}''$ belong to the plane defined by $\mathrm{P}^1,\mathrm{P}^2$, or: both $a)$ and $b)$.

Df.20 *Parallellity of Planes.*

$\boldsymbol{Pl}(\mathrm{P},\mathrm{P}') \parallel \boldsymbol{Pl}(\mathrm{P}'',\mathrm{P}''') . \equiv . \exists\,\boldsymbol{Lin}(\mathrm{P}^1,\mathrm{P}^2,\mathrm{P}^3) : \mathrm{P},\mathrm{P}',\mathrm{P}'',\mathrm{P}''' \in \boldsymbol{Lin}(\mathrm{P}^1,\mathrm{P}^2,\mathrm{P}^3)$

Translation: Two planes are said to be parallel **iff** their defining points belong to the same line.

Df.21 *Parallellity of Lines.*

$\boldsymbol{Lin}(\mathrm{P},\mathrm{P}',\mathrm{P}'') \parallel \boldsymbol{Lin}(\mathrm{P}^1,\mathrm{P}^2,\mathrm{P}^3) . \equiv . \exists\,\boldsymbol{Pl}(\mathrm{P}^a,\mathrm{P}^b) : \mathrm{P},\mathrm{P}',\mathrm{P}'',\mathrm{P}^1,\mathrm{P}^2,\mathrm{P}^3 \in \boldsymbol{Pl}(\mathrm{P}^a,\mathrm{P}^b)$

Translation: Two lines are said to be parallel **iff** their defining points belong to the same plane.

Df.22 *The Substrate.*

$\boldsymbol{Subst} \equiv \{\, \mathrm{P}^i \,|\, \forall\, \mathrm{P}^0, \mathrm{P}^1, \mathrm{P}^2, P_r^1, P_r^2, P_{r'}^2 :$

$1)\ \mathrm{P}^0\mathrm{P}^1(P_r^1) = \mathrm{P}^0\mathrm{P}^2(P_r^2) \;\Rightarrow\; \mathrm{P}^0\mathrm{P}^1 = \mathrm{P}^0\mathrm{P}^2 \,.\wedge.$

$2)\ \mathrm{P}^0\mathrm{P}^2(P_r^2) = \mathrm{P}^0\mathrm{P}^1\mathrm{P}^2(P_r^2) \;\Rightarrow\; \mathrm{P}^0\mathrm{P}^2 = \mathrm{P}^0\,\mathrm{P}^1\mathrm{P}^2 \,\}$

Translation: The Substrate is a set of points/particles characterized by the following properties: 1) if two points in the Substrate ever keep the same distance to a third one, they always do so; 2) if three points in the Substrate ever form an optical line, they always do so.

Comment: The Substrate thus conforms to the *Cosmological Principle* which is indispensable to the definability of a universal time and an absolute simultaneity; cf. Nicolas of Cusa [o.1450].

AX.4 *The Substrate is not Empty.*

$\exists\, \mathrm{P}, \exists\, \mathrm{P}' : \mathrm{P}, \mathrm{P}' \in \boldsymbol{Subst}$

Translation: There are at least two particles (observers) in the Substrate.

AX.5 *Coincidence leads to Collapse.*

$\forall\, \mathrm{P}, \forall\, \mathrm{P}', \forall\, \mathrm{P}'', \forall\, P_r'' : \mathrm{P}, \mathrm{P}', \mathrm{P}'' \in \boldsymbol{Subst} \;\Rightarrow .\; \mathrm{PP}''(P_r'') = 0 \Rightarrow \mathrm{P}'\mathrm{P}''(P_r'') = 0$

Translation: It holds for any three members of the Substrate that if two of them ever coincide at an instant they all coincide at that instant, so that the Substrate collapses into a single point.

AX.6 *Spheres in the Substrate.*

$\forall\, \mathrm{P}, \forall\, \mathrm{P}' : \mathrm{P}, \mathrm{P}' \in \boldsymbol{Subst} \;\Rightarrow .\; \exists\, \boldsymbol{S}(\mathrm{P}, \mathrm{P}') : \boldsymbol{S}(\mathrm{P}, \mathrm{P}') \subset \boldsymbol{Subst}$

Translation: For any two members of the Substrate there is a sphere defined by those two points which is a subset of the Substrate. There is no sphere which is not a subset of the Substrate.

AX.7 *Planes in the Substrate.*

$\forall\, \mathrm{P}, \forall\, \mathrm{P}' : \mathrm{P}, \mathrm{P}' \in \boldsymbol{Subst} \;\Rightarrow .\; \exists\, \boldsymbol{Pl}(\mathrm{P}, \mathrm{P}') : \boldsymbol{Pl}(\mathrm{P}, \mathrm{P}') \subset \boldsymbol{Subst}$

Translation: For any two members of the Substrate there is a plane defined by those two points which is a subset of the Substrate. There is no plane which is not a subset of the Substrate.

Df.23 *Fundamental Particles.*

$\mathrm{P} \in \boldsymbol{Subst} \,.\equiv.\, \mathrm{P} = \mathrm{P}_{FP}$

Translation: The particle P is called fundamental, P_{FP}, **iff** it belongs to the Substrate.

Df.24 *Accidental Particles.*

$\mathrm{Q} \notin \boldsymbol{Subst} \,.\equiv.\, \mathrm{Q} = \mathrm{Q}_{\boldsymbol{AP}}$

Translation: The particle Q is called accidental, $\mathrm{Q}_{\boldsymbol{AP}}$, **iff** it does not belong to the Substrate.

AX.8 *Existence of Accidental Particles.*

$\exists\,\mathrm{Q} : \mathrm{Q} \notin \boldsymbol{Subst} \wedge \forall\,\mathrm{Q}, Q_r : \exists\,\mathrm{P} : \mathrm{P} \in \boldsymbol{Subst} \wedge \mathrm{PQ}(Q_r) = 0$

Translation: There are accidental particles, and for any such particle there is at any instant a fundamental particle with which it momentarily coincides and which thus defines its position.

AX.9 *Space has Three Dimensions.*

$\forall\,\mathrm{P}', \forall\,\mathrm{P}'', \forall\,\mathrm{P}''' : \mathrm{P}', \mathrm{P}'', \mathrm{P}''' \in \boldsymbol{Subst} \wedge \mathrm{P}'\mathrm{P}'' = \mathrm{P}''\mathrm{P}''' = \mathrm{P}'''\mathrm{P}'$

$\Rightarrow\; : \exists\,2\,\mathrm{P} : \mathrm{P} \in \boldsymbol{Subst} \wedge \mathrm{PP}' = \mathrm{PP}'' = \mathrm{PP}''' = \mathrm{P}'\mathrm{P}''$

Translation: For any three equidistant particles, members of the Substrate, there are two and only two more particles, members of the Substrate, keeping the same distance to the other three.
Comment: The difference between the two tetrahedra is the origin af handedness in 3-space.

Df.25 *Space is Flat (Euclidean).*

$\forall\,\mathrm{P}^0, \forall\,\mathrm{P}^1, \forall\,\mathrm{P}^2, \forall\,\mathrm{P}^3, \forall\,\mathrm{P}^4, \forall\,\mathrm{P}^5, \forall\,\mathrm{P}^6 : \mathrm{P}^0, \mathrm{P}^1, \mathrm{P}^2, \mathrm{P}^3, \mathrm{P}^4, \mathrm{P}^5, \mathrm{P}^6 \in \boldsymbol{Subst}$

$\Rightarrow\; : \mathrm{P}^0\mathrm{P}^1 = \mathrm{P}^0\mathrm{P}^2 = \mathrm{P}^0\mathrm{P}^3 = \mathrm{P}^0\mathrm{P}^4 = \mathrm{P}^0\mathrm{P}^5 = \mathrm{P}^0\mathrm{P}^6 = \mathrm{P}^1\mathrm{P}^2 = \mathrm{P}^2\mathrm{P}^3 = \mathrm{P}^3\mathrm{P}^4 = \mathrm{P}^4\mathrm{P}^5 = \mathrm{P}^5\mathrm{P}^6$

$\Rightarrow\; .\ \exists\,\boldsymbol{Pl}(\mathrm{P}, \mathrm{P}') : \boldsymbol{Pl}(\mathrm{P}, \mathrm{P}') \subset \boldsymbol{Subst} \;\wedge\; \mathrm{P}^0, \mathrm{P}^1, \mathrm{P}^2, \mathrm{P}^3, \mathrm{P}^4, \mathrm{P}^5, \mathrm{P}^6 \in \boldsymbol{Pl}(\mathrm{P}, \mathrm{P}')$

Translation: Space is flat, or Euclidean, **iff** for all particles $\mathrm{P}^0, \mathrm{P}^1, \mathrm{P}^2, \mathrm{P}^3, \mathrm{P}^4, \mathrm{P}^5, \mathrm{P}^6$, members of the Substrate, if $\mathrm{P}^1, \mathrm{P}^2, \mathrm{P}^3, \mathrm{P}^4, \mathrm{P}^5, \mathrm{P}^6$ form a regular hexagon with P^0 in its center, all distances between neighbouring corners being equal to their distances (radii) from the center, then there is a plane, subset of the Substrate, to which they all belong, the six corners as well as their center.
Comment: To begin with the question is left open whether space is Euclidean or non-Euclidean. If it is non-Euclidean and hyperbolic, the sum of the sides of the hexagon is more than six radii. If it is non-Euclidean and spherical, the sum of the sides of the hexagon is less than six radii.

Df.26 *Midway Particles.*

$\mathrm{P} = \boldsymbol{m}(\mathrm{P}^1, \mathrm{P}^2) \,.\equiv.\, \mathrm{P}^1\mathrm{P}^2 = \mathrm{P}^1\mathrm{PP}^2 \wedge \mathrm{PP}^1 = \mathrm{PP}^2$

Translation: P is the midway particle of P^1, P^2 **iff** $\mathrm{P}^1\mathrm{P}^2$ equals $\mathrm{P}^1\mathrm{PP}^2$ and PP^1 equals PP^2.

Th.11 *There are Midway Particles.*

$\forall\,\mathrm{P}^1, \forall\,\mathrm{P}^2 : \boldsymbol{Pl}(\mathrm{P}^1, \mathrm{P}^2) \subset \boldsymbol{Subst} \,.\Rightarrow \exists\,\mathrm{P} : \mathrm{P} = \boldsymbol{m}(\mathrm{P}^1, \mathrm{P}^2) \in \boldsymbol{Subst}$

Translation: Any two fundamental particles have a midway particle which is also fundamental.

Proof: $\forall\,\mathrm{P}, \forall\,\mathrm{P}^1, \forall\,\mathrm{P}^2 : \boldsymbol{Pl}(\mathrm{P}^1, \mathrm{P}^2) \subset \boldsymbol{Subst} \,.\Rightarrow\; :$ cf. Ax.7

$\mathrm{P} = \boldsymbol{m}(\mathrm{P}^1, \mathrm{P}^2) \,.\Rightarrow.\, \mathrm{P} \in \boldsymbol{Pl}(\mathrm{P}^1, \mathrm{P}^2) \subset \boldsymbol{Subst}$

Df.27 *Center Particles.*

$\mathrm{P} = \boldsymbol{c}(\mathrm{P}^1, \mathrm{P}^2, \mathrm{P}^3) \,.\equiv\; :\; \mathrm{P}^2, \mathrm{P}^3 \in \boldsymbol{C}(\mathrm{P}, \mathrm{P}^1)$

Translation: P is the center particle of P^1, P^2, P^3 **iff** $\mathrm{P}^2, \mathrm{P}^3$ belongs to the circle defined by P, P^1.

Th.12 *There are Center Particles.*

$\forall \mathrm{P}^1, \forall \mathrm{P}^2, \forall \mathrm{P}^3 : \boldsymbol{Lin}(\mathrm{P}^1, \mathrm{P}^2, \mathrm{P}^3) \subset \boldsymbol{Subst} \,.\Rightarrow\, : \exists\, \mathrm{P} : \mathrm{P} = \boldsymbol{c}(\mathrm{P}^1, \mathrm{P}^2, \mathrm{P}^3) \in \boldsymbol{Subst}$

Translation: If three fundamental particles define a line, and thus are not themselves in line, they have a center particle which is also fundamental.

Proof: $\forall \mathrm{P}, \forall \mathrm{P}^1, \forall \mathrm{P}^2, \forall \mathrm{P}^3 : \boldsymbol{Lin}(\mathrm{P}^1, \mathrm{P}^2, \mathrm{P}^3) \subset \boldsymbol{Subst} \,.\Rightarrow\, :$

$\mathrm{P} = \boldsymbol{c}(\mathrm{P}^1, \mathrm{P}^2, \mathrm{P}^3) \,.\Rightarrow.\, \mathrm{P} \in \boldsymbol{Lin}(\mathrm{P}^1, \mathrm{P}^2, \mathrm{P}^3) \subset \boldsymbol{Subst}$

AX.10 *Signals follow Optical Lines in the Substrate.*

$\forall Q'_e, \forall Q''_a : Q'_e \in \mathrm{Q}' \wedge Q''_a \in \mathrm{Q}'' \wedge Q'_e \to Q''_a \,.\Rightarrow\, : \exists\, \mathrm{P}, \exists\, \mathrm{P}', \exists\, \mathrm{P}'' : \mathrm{P}, \mathrm{P}', \mathrm{P}'' \in \boldsymbol{Subst}\,.$

$.\wedge.\, \mathrm{P}'\mathrm{Q}'(Q'_e) = \mathrm{P}''\mathrm{Q}''(Q''_a) = 0 \wedge \mathrm{P} = \boldsymbol{m}(\mathrm{P}', \mathrm{P}'') \wedge \exists\, P_t : P_t \in \mathrm{P} \wedge Q'_e \to P_t \to Q''_a$

Translation: If a signal emitted from Q′ at the instant Q'_e when Q′ coincided with P′, member of the Substrate, was absorbed by Q″ at the instant Q''_a when Q″ coincided with P″, also member of the Substrate, it must have passed P, midway point of P′ and P″, at some instant P_t.

Comment: The Substrate thus functions as an Aether, i.e. as if it were a material medium for the propagation of signals. Signals are propagated along direct tracks and with the same speed.

Df.28 *Relative Simultaneity of Events.*

$\boldsymbol{sim}_{\mathrm{P}}(P'_r, P''_r) \,.\equiv.\, \exists\, \mathrm{P}, \mathrm{P}', \mathrm{P}'' : \mathrm{P}, \mathrm{P}', \mathrm{P}'' \in \boldsymbol{Subst} \wedge \mathrm{PP}'(P'_r) = \mathrm{PP}''(P''_r)$

Translation: Two events, P'_r in P′, P''_r in P″, are said to be P-simultaneous relative to observer P **iff** P, P′, P″ all belong to the Substrate and P′ at P'_r is equidistant with P″ at P''_r relative to P.

Th.13 *Simultaneity is Reciprocal with Respect to any Point on a Plane.*

$\forall P'_r, \forall P''_r : \mathrm{P} \in \boldsymbol{Pl}(\mathrm{P}', \mathrm{P}'') \,.\Rightarrow\, : \boldsymbol{sim}_{\mathrm{P}}(P'_r, P''_r) \Leftrightarrow \boldsymbol{sim}_{\mathrm{P}}(P''_r, P'_r)$

Translation: The event P'_r is P-simultaneous with the event P''_r **iff** P''_r is P-simultaneous with P'_r.

Proof: $\mathrm{P} \in \boldsymbol{Pl}(\mathrm{P}', \mathrm{P}'') \,.\Rightarrow\, : \mathrm{PP}' = \mathrm{PP}'' \Rightarrow .\, \mathrm{PP}'(P'_r) = \mathrm{PP}''(P''_r)$

Cor.4 *Simultaneity is Reciprocal with Respect to a Midway Particle.*

$\forall P'_r, \forall P''_r : \mathrm{P} = \boldsymbol{m}(\mathrm{P}', \mathrm{P}'') \,.\Rightarrow\, : \boldsymbol{sim}_{\mathrm{P}}(P'_r, P''_r) \Leftrightarrow \boldsymbol{sim}_{\mathrm{P}}(P''_r, P'_r)$

Translation: The event P'_r is P-simultaneous with the event P''_r **iff** P''_r is P-simultaneous with P'_r.

Proof: $\mathrm{P} = \boldsymbol{m}(\mathrm{P}', \mathrm{P}'') \in \boldsymbol{Pl}(\mathrm{P}', \mathrm{P}'') \,.\Rightarrow\, : \mathrm{PP}' = \mathrm{PP}'' \Rightarrow .\, \mathrm{PP}'(P'_r) = \mathrm{PP}''(P''_r)$

Th.14 *Simultaneity is Transitive with Respect to any Point on a Line.*

$\forall P'_r, \forall P''_r, \forall P'''_r : \mathrm{P} \in \boldsymbol{Lin}(\mathrm{P}', \mathrm{P}'', \mathrm{P}''') \subset \boldsymbol{Subst} \,.\Rightarrow\, :$

$\boldsymbol{sim}_{\mathrm{P}}(P'_r, P''_r) \wedge \boldsymbol{sim}_{\mathrm{P}}(P''_r, P'''_r) \,.\Rightarrow\, \boldsymbol{sim}_{\mathrm{P}}(P'_r, P'''_r)$

Translation: Simultaneity is transitive between pairs of events relative to points on a line.

Comment: The condition is that the particles P′, P″, P‴ do not themselves belong to a line.

Proof: $\mathrm{P} \in \boldsymbol{Lin}(\mathrm{P}', \mathrm{P}'', \mathrm{P}''') \,.\Rightarrow.\, \mathrm{PP}' = \mathrm{PP}'' = \mathrm{PP}'''$

Cor.5 *Simultaneity is Transitive with Respect to a Center Particle.*

$\forall P'_r, \forall P''_r, \forall P'''_r : \mathrm{P} = \boldsymbol{c}(\mathrm{P}', \mathrm{P}'', \mathrm{P}''') \subset \boldsymbol{Subst} \,.\Rightarrow\, :$

$\boldsymbol{sim}_{\mathrm{P}}(P'_r, P''_r) \wedge \boldsymbol{sim}_{\mathrm{P}}(P''_r, P'''_r) \,.\Rightarrow \boldsymbol{sim}_{\mathrm{P}}(P'_r, P'''_r)$

Translation: Simultaneity is transitive between pairs of events relative to a center particle P.
Comment: The condition is that the particles $\mathrm{P}', \mathrm{P}'', \mathrm{P}'''$ do not themselves belong to a line.
Proof: $\mathrm{P} = \boldsymbol{c}(\mathrm{P}', \mathrm{P}'', \mathrm{P}''') \in \boldsymbol{Lin}(\mathrm{P}', \mathrm{P}'', \mathrm{P}''') \,.\Rightarrow .\, \mathrm{PP}' = \mathrm{PP}'' = \mathrm{PP}'''$

Th.15 *Simultaneity is Transitive between Particles on a Sphere.*

$\forall \mathrm{P}', \forall \mathrm{P}'', \forall \mathrm{P}''' : \exists \mathrm{P} : \mathrm{P}'', \mathrm{P}''' \in \boldsymbol{S}(\mathrm{P}, \mathrm{P}') \subset \boldsymbol{Subst} \,.\Rightarrow .$

$\boldsymbol{sim}_{\mathrm{P}}(P'_r, P''_r) \wedge \boldsymbol{sim}_{\mathrm{P}}(P''_r, P'''_r) \,.\Rightarrow .\, \boldsymbol{sim}_{\mathrm{P}}(P'_r, P'''_r)$

Translation: Simultaneity is transitive for all particles on a sphere, relative to its center P.
Comment: The condition is that the particles $\mathrm{P}', \mathrm{P}'', \mathrm{P}'''$ do not themselves belong to a line.
Proof: $\mathrm{P}'', \mathrm{P}''' \in \boldsymbol{S}(\mathrm{P}, \mathrm{P}') \,.\Rightarrow .\, \mathrm{PP}' = \mathrm{PP}'' = \mathrm{PP}'''$

Th.16 *Simultaneity is Transitive between Equidistant Particles on a Line.*

$\forall \mathrm{P}^0, \forall \mathrm{P}^1, \forall \mathrm{P}^{-1}, \forall \mathrm{P}^2, \forall \mathrm{P}^{-2}, \forall \mathrm{P}^{n-1}, \forall \mathrm{P}^{1-n}, \forall \mathrm{P}^n, \forall \mathrm{P}^{-n} : \exists \boldsymbol{Lin}(\mathrm{P}', \mathrm{P}'', \mathrm{P}''') :$

$\mathrm{P}^0, \mathrm{P}^1, \mathrm{P}^{-1}, \mathrm{P}^2, \mathrm{P}^{-2}, \mathrm{P}^{n-1}, \mathrm{P}^{1-n}, \mathrm{P}^n, \mathrm{P}^{-n} \in \boldsymbol{Lin}(\mathrm{P}', \mathrm{P}'', \mathrm{P}''') \subset \boldsymbol{Subst} \,.\Rightarrow .:$

$\mathrm{P}^{-n}\mathrm{P}^{1-n} = \ldots = \mathrm{P}^{-2}\mathrm{P}^{-1} = \mathrm{P}^{-1}\mathrm{P}^0 = \mathrm{P}^0\mathrm{P}^1 = \mathrm{P}^1\mathrm{P}^2 = \ldots = \mathrm{P}^{n-1}\mathrm{P}^n \,.\Rightarrow\, :$

$\boldsymbol{sim}_{\mathrm{P}^0}(P_1^{-1}, P_1^1) \wedge \boldsymbol{sim}_{\mathrm{P}^0}(P_2^{-2}, P_2^2) \wedge \boldsymbol{sim}_{\mathrm{P}^0}(P_2^{-2}, P_2^0) \wedge \boldsymbol{sim}_{\mathrm{P}^0}(P_2^0, P_2^2) \wedge$
$\boldsymbol{sim}_{\mathrm{P}^0}(P_3^{-3}, P_3^3) \wedge \boldsymbol{sim}_{\mathrm{P}^0}(P_3^{-3}, P_3^{-1}) \wedge \boldsymbol{sim}_{\mathrm{P}^0}(P_3^{-1}, P_3^1) \wedge \boldsymbol{sim}_{\mathrm{P}^0}(P_3^1, P_3^3) \wedge \boldsymbol{sim}_{\mathrm{P}^0}(P_4^{-4}, P_4^4) \wedge$
$\boldsymbol{sim}_{\mathrm{P}^0}(P_4^{-4}, P_4^{-2}) \wedge \boldsymbol{sim}_{\mathrm{P}^0}(P_4^{-2}, P_4^0) \wedge \boldsymbol{sim}_{\mathrm{P}^0}(P_4^0, P_4^2) \wedge \boldsymbol{sim}_{\mathrm{P}^0}(P_4^2, P_4^4) \wedge \boldsymbol{sim}_{\mathrm{P}^0}(P_4^{-2}, P_4^2) \wedge \ldots$

Translation: Simultaneity is transitive for all equidistant particles on a line relative to some arbitrarily chosen midway particle P^0, counting right units as positive and left units as negative.
Proof: $\mathrm{P}^0\mathrm{P}^{-1}(P_1^{-1}) = \mathrm{P}^0\mathrm{P}^1(P_1^1) \wedge \mathrm{P}^0\mathrm{P}^{-1}\mathrm{P}^{-2}(P_2^{-2}) = \mathrm{P}^0\mathrm{P}^{-1}\mathrm{P}^0(P_2^0) = \mathrm{P}^0\mathrm{P}^1\mathrm{P}^0(P_2^0) = \mathrm{P}^0\mathrm{P}^1\mathrm{P}^2(P_2^2) \wedge$
$\mathrm{P}^0\mathrm{P}^{-1}\mathrm{P}^{-2}\mathrm{P}^{-3}(P_3^{-3}) = \mathrm{P}^0\mathrm{P}^{-1}\mathrm{P}^{-2}\mathrm{P}^{-1}(P_3^{-1}) = \mathrm{P}^0\mathrm{P}^{-1}\mathrm{P}^0\mathrm{P}^{-1}(P_3^{-1}) = \mathrm{P}^0\mathrm{P}^1\mathrm{P}^0\mathrm{P}^1(P_3^1) = \mathrm{P}^0\mathrm{P}^1\mathrm{P}^2\mathrm{P}^1(P_3^1) = \mathrm{P}^0\mathrm{P}^1\mathrm{P}^2\mathrm{P}^3(P_3^3) \wedge$
$\mathrm{P}^0\mathrm{P}^{-1}\mathrm{P}^{-2}\mathrm{P}^{-3}\mathrm{P}^{-4}(P_4^{-4}) = \mathrm{P}^0\mathrm{P}^{-1}\mathrm{P}^{-2}\mathrm{P}^{-3}\mathrm{P}^{-2}(P_4^{-2}) = \mathrm{P}^0\mathrm{P}^{-1}\mathrm{P}^0\mathrm{P}^{-1}\mathrm{P}^0(P_4^0) = \mathrm{P}^0\mathrm{P}^1\mathrm{P}^0\mathrm{P}^1\mathrm{P}^0(P_4^0) = \mathrm{P}^0\mathrm{P}^1\mathrm{P}^2\mathrm{P}^3\mathrm{P}^2(P_4^2) = \mathrm{P}^0\mathrm{P}^1\mathrm{P}^2\mathrm{P}^3\mathrm{P}^4(P_4^4)$
Comment: By inserting more and more midway points we can refine the partitioning to pursue the construction of a network which can catch any particle on the line with the wanted precision.

Scholium. Combining theorems 13-16, simultaneity is definable for the entire Substrate. However only a finite number of the points contained in a line can be covered by our procedure. In order to ensure simultaneity for all points in a line we are therefore faced with the choice between postulating atoms of time, thus also of space, or postulating simultaneity to be absolute. It turns out, however, that assuming simultaneity to be absolute does not prevent moving clocks from becoming retarded, as evidenced by experiment and in agreement with standard relativity. The condition is that our concepts of rest and motion are defined relative to the Substrate.

AX.11 *Simultaneity is Absolute in the Substrate.*
$\forall \mathrm{P}, \forall \mathrm{P}', \forall \mathrm{P}'', \forall \mathrm{P}''' : \mathrm{P}, \mathrm{P}', \mathrm{P}'', \mathrm{P}''' \in \boldsymbol{Subst} \,.\Rightarrow\; :$
$\exists P_r', \exists P_r'', \exists P_r''' : \boldsymbol{sim}_{\mathrm{P}}(P_r', P_r'') \wedge \boldsymbol{sim}_{\mathrm{P}}(P_r'', P_r''') \wedge \boldsymbol{sim}_{\mathrm{P}}(P_r', P_r''')$

Translation: Any triple of particles, members of the Substrate, contain events that are pairwise simultaneous relative to any other particle also belonging to the Substrate.
Comment: As a consequence hereof there are no points on a line which are not simultaneous. We shall therefore consider a line as being continuous at any instant.

Cor.6 *Simultaneity is Transitive in the Substrate.*
$\forall \mathrm{P}, \forall \mathrm{P}', \forall \mathrm{P}'', \forall \mathrm{P}''' : \mathrm{P}, \mathrm{P}', \mathrm{P}'', \mathrm{P}''' \in \boldsymbol{Subst} \,.\Rightarrow\; :$
$\exists P_r', \exists P_r'', \exists P_r''' : \boldsymbol{sim}_{\mathrm{P}}(P_r', P_r'') \wedge \boldsymbol{sim}_{\mathrm{P}}(P_r'', P_r''') \,.\Rightarrow .\, \boldsymbol{sim}_{\mathrm{P}}(P_r', P_r''')$

Translation: Simultaneity is transitive for all members of the Substrate.
Proof: Follows immediately from the preceding axiom and QT.

Cor.7 *Simultaneity is Transitive in the whole Universe.*
$\forall \mathrm{P}, \forall \mathrm{Q}', \forall \mathrm{Q}'', \forall \mathrm{Q}''' : \mathrm{P} \in \boldsymbol{Subst} \wedge \mathrm{Q}', \mathrm{Q}'', \mathrm{Q}''' \in \boldsymbol{Universe} \,.\Rightarrow\; :$
$\exists Q_r', \exists Q_r'', \exists Q_r''' : \boldsymbol{sim}_{\mathrm{P}}(Q_r', Q_r'') \wedge \boldsymbol{sim}_{\mathrm{P}}(Q_r'', Q_r''') \,.\Rightarrow .\, \boldsymbol{sim}_{\mathrm{P}}(Q_r', Q_r''')$

Translation: Simultaneity is transitive for all members of the entire Universe.
Proof: Follows immediately from the preceding theorem, axiom 8, and df.3.

Df.29 *Zigzag Signals.*
$\boldsymbol{zs}(\mathrm{P}, \mathrm{P}') \,.\equiv .\, \exists P_n, P_n' : P_n \in \mathrm{P} \wedge P_n' \in \mathrm{P}' \wedge P_0 \to P_0' \to P_1 \to P_1' \to P_2 \to P_2' \ldots$

Translation: There was a zigzag signal between P and P′ **iff** there were P_n in P and P_n' in P′ forming a chain of events $P_0, P_0', P_1, P_1', P_2, P_2'$ following each other in temporal succession.

Df.30 *Cosmic Clocks.*
$\boldsymbol{ccl}(\mathrm{PP}') \,.\equiv .\, \exists \boldsymbol{zs}(\mathrm{P}, \mathrm{P}') : \mathrm{P}, \mathrm{P}' \in \boldsymbol{Subst} \wedge$
$\{\ldots, P_0, P_1, P_2, \ldots\} \rightharpoonup \{\ldots, T(P_0), T(P_1), T(P_2), \ldots\} \subseteq \boldsymbol{\mathcal{R}}$

Translation: The cosmic clock carried by P **is** a Langevin clock with unit PP′P defined by a zigzag signal exchanged between P & P′, P & P′ being members of the Substrate and successive events P_i being mapped on instants $T(P_i)$ constituting a subset of the set of real numbers $\boldsymbol{\mathcal{R}}$.

Th.17 *There are Cosmic Clocks.*
$\forall \mathrm{P}, \forall \mathrm{P}' : \mathrm{P}, \mathrm{P}' \in \boldsymbol{Subst} \,.\Rightarrow\; : \exists\, \boldsymbol{ccl}(\mathrm{PP}')$

Translation: For any pair of fundamental particles we can construct a cosmic clock.
Proof: Follows immediately from the definition of cosmic clocks and ax.1.

Df.31 *Non-Identity of Cosmic Clocks.*
$\boldsymbol{ccl}(\mathrm{PP'}) \neq \boldsymbol{ccl}(\mathrm{P'P})$
Translation: Two cosmic clocks are different **iff** their carriers are different.

Df.32 *Similarity of Cosmic Clocks.*
$\boldsymbol{ccl}(\mathrm{PP'}) \simeq \boldsymbol{ccl}(\mathrm{P''P'''}) \,.\equiv.\, \mathrm{PP'} = \mathrm{P''P'''}$
Translation: Two cosmic clocks are similar **iff** their unit distances are equal.

Cor.8 *Similarity of Cosmic Clocks is Reciprocal.*
$\forall\,\mathrm{P}, \forall\,\mathrm{P'} : \boldsymbol{ccl}(\mathrm{PP'}) \simeq \boldsymbol{ccl}(\mathrm{P'P})$
Translation: Two different cosmic clocks defined by the same pair of particles are similar.
Proof: Follows immediately from their unit distances being equal.

Cor.9 *Similarity of Cosmic Clocks is Transitive.*
$\forall\,\mathrm{P}, \forall\,\mathrm{P'}, \forall\,\mathrm{P''}, \forall\,\mathrm{P'''} :$
$$\boldsymbol{ccl}(\mathrm{PP'}) \simeq \boldsymbol{ccl}(\mathrm{P'P''}) \wedge \boldsymbol{ccl}(\mathrm{P'P''}) \simeq \boldsymbol{ccl}(\mathrm{P''P'''}) \,.\Rightarrow.\, \boldsymbol{ccl}(\mathrm{PP'}) \simeq \boldsymbol{ccl}(\mathrm{P''P'''})$$
Translation: Similarity is transitive between any triple of cosmic clocks.
Proof: Follows immediately from their unit distances being equal.

Df.33 *Signalfunctions.*
$T^{\mathrm{P'}} = \Theta^{\mathrm{PP'}}(T^{\mathrm{P}}).\equiv.\, T(P'_i) = \Theta^{\mathrm{PP'}}(T(P_{i-1})).\equiv.\, \exists\, \boldsymbol{zs}(\mathrm{P},\mathrm{P'}) : T(P_i) \rightharpoonup T(P'_j)$
Translation: There is a signalfunction from instants $T(P_{i-1})$ on the clock $\boldsymbol{ccl}(\mathrm{PP'})$ carried by P to instants $T(P'_{\mathrm{i}})$ on the clock $\boldsymbol{ccl}(\mathrm{P'P})$ carried by P′ **iff** there is a zigzag signal between P & P′ together with a mapping of preceding instants $T(P_{i-1})$ in P to succeding instants $T(P'_{\mathrm{i}})$ in P′.

Df.34 *Mapping a Clock onto Itself.*
$\boldsymbol{ccl}(\mathrm{PP'}) \rightharpoonup \boldsymbol{ccl}(\mathrm{PP'}) \,.\equiv.\, T_i^{\mathrm{P}} = \Theta^{\mathrm{P'P}}\Theta^{\mathrm{PP'}}(T_{i-1}^{\mathrm{P}}).\equiv.\, T(P_i) = \Theta^{\mathrm{P'P}}\big(\Theta^{\mathrm{PP'}}(T(P_{i-1}))\big)$
Translation: There is a mapping of the clock $\boldsymbol{ccl}(\mathrm{PP'})$ onto itself **iff** there is a zigzag signal from P to P′ and back together with a signalfunction from P to P′ mapping instants in P onto instants in P′ and another signalfunction from P′ to P mapping instants in P′ onto instants in P.

Df.35 *Congruence of Cosmic Clocks.*
$\boldsymbol{ccl}(\mathrm{PP'}) \equiv \boldsymbol{ccl}(\mathrm{P'P}) \,.\equiv.\, \Theta^{\mathrm{P'P}} = \Theta^{\mathrm{PP'}} = \Theta \,.\equiv.\, \Theta^{\mathrm{P'P}}\Theta^{\mathrm{PP'}} = \Theta\Theta$
Translation: The clock $\boldsymbol{ccl}(\mathrm{PP'})$ of P is said to be congruent with the clock $\boldsymbol{ccl}(\mathrm{P'P})$ of P′ **iff** the signalfunction from P to P′ is identical to the signalfunction from P′ to P.

Comment: Clocks are made congruent by regraduation, i.e., a formal adjustment of units & zero. A method for deriving the functional square root of $\Theta^{P'P}\Theta^{PP'}$ was found by Milne & Whitrow. That congruence of cosmic clocks is reciprocal follows at once from the definition just given. However, it was also shown by Whitrow & Milne that congruence of clocks is transitive among collinear particles iff their signalfunctions commute. We shall go no further into this here.

Th.18 *Congruent Cosmic Clocks are Similar.*

$\forall\, \mathrm{P}, \forall\, \mathrm{P'} : \boldsymbol{ccl}(\mathrm{PP'}) \equiv \boldsymbol{ccl}(\mathrm{P'P})\,.\Rightarrow.\,\boldsymbol{ccl}(\mathrm{PP'}) \simeq \boldsymbol{ccl}(\mathrm{P'P})$

Translation: If two cosmic clocks are congruent they are also similar, i.e., go at the same rate.
Comment: Similar clocks keep the same rate. Congruent clocks also have a common zero.

Df.36 *Atomic Clocks.*

$\boldsymbol{acl}(\mathrm{Q}, r_a)\,.\equiv.\,\mathrm{Q} \in \boldsymbol{Universe} \wedge unit = r_a \wedge$

$\{..., Q_0, Q_1, Q_2, ...\} \rightharpoonup \{..., t(Q_0), t(Q_1), t(Q_2), ...\} \subseteq \boldsymbol{\mathcal{Z}}_{\pm}$

Translation: The atomic clock carried by Q **is** a mechanism devised to amplify the oscillations of an atom of specified type displaying the natural frequency $1/r_a$, events P_i being mapped on instants $T(P_i)$ constituting a very fine-grained subset of the set of natural numbers $\boldsymbol{\mathcal{Z}}_{\pm}$.

AX.12 *There are Atomic Clocks.*

$\forall\, \mathrm{Q} : \mathrm{Q} \in \boldsymbol{Universe}\,.\Rightarrow.\,\exists\, \boldsymbol{acl}(\mathrm{Q}, r_a)$

Translation: Any point or particle in the universe is the carrier of at least one atomic clock.
Comment: This axiom, or postulate, has a somewhat different status, as compared to the other eleven, since it involves conditions of a more physical (not purely geometrical) character.

Df.37 *Atomic Master Clocks.*

$\boldsymbol{mcl}(\mathrm{P}, r_a)\,.\equiv.\,\boldsymbol{acl}(\mathrm{P}, r_a) \wedge \mathrm{P} = \mathrm{P}_{FP} \in \boldsymbol{Subst}$

Translation: A master clock **is** an atomic clock carried by a fundamental particle.

Df.38 *Atomic Slave Clocks.*

$\boldsymbol{scl}(\mathrm{Q}, r_a)\,.\equiv.\,\boldsymbol{acl}(\mathrm{Q}, r_a) \wedge \mathrm{Q} = \mathrm{Q}_{AP} \notin \boldsymbol{Subst}$

Translation: A slave clock **is** an atomic clock carried by an accidental particle.

Df.39 *Similarity of Atomic Master Clocks.*

$\boldsymbol{mcl}(\mathrm{P}, r_a) \simeq \boldsymbol{mcl}(\mathrm{P'}, r'_a)\,.\equiv.\,r_a = r'_a = r_o$

Translation: Two atomic master clocks are similar (i.e., keep the same rate) **iff** they are both controlled by atoms of the same type displaying the same frequencies and having the same radii.
Comment: Only the master clocks of fundamental particles can be similar in this sense.

Df.40 *A Stationary Universe*

$\boldsymbol{Universe}$ *stationary* . ≡ . $\forall\, \boldsymbol{mcl}(\mathrm{P}, r_a), \exists\, \boldsymbol{ccl}(\mathrm{PP}') : \Delta t / \Delta T \equiv const.$

Translation: The universe is stationary **iff** for any atomic master clock with the natural rate $1/r_a = 1/\Delta t$ there is a cosmic clock keeping the rate $1/\mathrm{PP}' = 1/\Delta T$ so that $\Delta t/\Delta T = const.$

Comment: This is equivalent to saying that the radii r_a of certain atoms are constant compared to some arbitrarily chosen cosmic distance PP′. Thus there is only one natural scale of time.

Df.41 *An Expanding Universe*

$\boldsymbol{Universe}$ *expanding* . ≡ . $\forall\, \boldsymbol{mcl}(\mathrm{P}, r_a), \exists\, \boldsymbol{ccl}(\mathrm{PP}') : \Delta t / \Delta T \equiv \mathcal{R}(t) \equiv \mathcal{S}(T)$

Translation: The universe is stationary **iff** for any atomic master clock with the natural rate $1/\Delta t$ there is a cosmic clock keeping the rate $1/\Delta T$ so that $\Delta t/\Delta T = \mathcal{R}(t) = \mathcal{S}(T) \rightarrow \infty$

Comment: This is equivalent to saying that the radii of atoms are steadily shrinking as compared to some arbitrarily chosen cosmic distance. So there are two natural time scales.

Df.42 *Frames Associated with Fundamental Particles* .

$\boldsymbol{Fra}(\mathrm{P})$. ≡ . $\exists\, \{\mathrm{Q}_i \,|\, \mathrm{P} = \mathrm{P}_{FP} \wedge \mathrm{PQ}_i = k_i\, r_a \wedge \boldsymbol{mcl}(\mathrm{P}, r_a) \underline{\equiv} \boldsymbol{scl}(\mathrm{Q}, r_a)\}$

Translation: The fundamental particle P is associated with a stationary reference frame **iff** P is surrounded by accidental particles Q keeping a fixed distance to P as origo, counted by a finite number of standard atomic radii, each of these particles being provided with atomic slave clocks which are kept congruent to the original master clock of P and situated in origo of P's frame.

Comment: If the universe is stationary, all such reference frames coincide with the Substrate. If, by contrast, the universe is expanding, it is a main point of the present axiomatization that there are no such frames in nature, and that their artificial construction would necessitate that the involved accidental particles were constrained by external forces to remain in their positions at fixed distances from the origo and their associated slave clocks were constrained by external forces to remain congruent with the master clock placed in origo.

Df.43 *Frames Associated with Accidental Particles* .

$\boldsymbol{fra}(\mathrm{Q})$. ≡ . $\exists\, \{\mathrm{R}_i \,|\, \mathrm{Q} = \mathrm{Q}_{AP} \wedge \mathrm{QR}_i = k_i\, r_a \wedge \boldsymbol{scl}(\mathrm{Q}, r_a) \underline{\equiv} \boldsymbol{scl}(\mathrm{R}, r_a)\}$

Translation: The accidental particle Q is associated with a comoving reference frame **iff** Q is followed by other accidental particles keeping a fixed distance to Q as origo, counted by a finite number of standard atomic radii, each of these particles being provided with atomic slave clocks which are kept congruent to the presumed master clock of Q and situated in origo of Q's frame.

Comment: If the universe is stationary, all such reference frames coincide with the Substrate. If, by contrast, the universe is expanding, it is our view that there are no such frames in nature, and that they can only be devised by imposing artificial constraints.

§3. *CONCLUSION*

Although our formal presentation is exceedingly sketchy, I think it allows us to affirm the conclusion of Walker [1959], that it is possible to assign clocks to all particles in the Substrate "which are not merely *congruent*, but also *equivalent* to each other", so that we henceforth have "the product structure $\mathcal{T} \times \mathcal{C}$ on the set of all events" (i.e. a *time-space* in the sense of Mercier, MW), $\mathcal{T}$ being a temporal parameter - *a cosmic time* - and $\mathcal{C}$ being the space of particles.

However, Walker considered only one type of clocks, viz. those we have called cosmic. Further, Walker considered a single Substrate only, the Substrate being comparable to a unique "spray" in the sense of Schutz [1973]. We shall here follow Milne and Walker by assuming the Substrate to be singular and unique. However, by introducing the atomic master clocks of fundamental particles we have, in contrast to Walker, also considered another type of clocks. This move is essential, since a comparison of cosmic and atomic clocks is necessary for making sense of an answer to the question whether the universe is stationary or expanding.

According to Eddington, "the theory of the expanding cosmos is equivalent to the theory of the shrinking atom". So it is impossible to decide whether the universe is expanding relative to the fixed sizes of its contents or whether its contents are shrinking relative to the dimensions of their stationary surroundings. It is obvious that different types of clocks, cosmic and atomic, are bound to measure spatial distances in different ways. This shows that Milne was right when he insisted that the distinction between two basic scales of time, T & t, is mandatory to physics. Now the principle of inertia, if valid at all, can only hold good relative to one of these scales. We shall follow Milne by assuming that it holds relative to the cosmic T-scale. Consequently we may expect that the free motion of particles is exposed to spontaneous accelerations when described relative to the atomic t-scale. This, after all, might be the cause of gravitation.

In fact, the phenomenon of gravitation can only emerge within an expanding universe. In such a universe the principle of Mach would finally be vindicated. However, it would hold the other way round than presumed by Mach and Einstein, as well as by their host of adherents. By this turn gravity would be explained by inertia, instead of inertia being explained by gravity. The Substrate, as described according to the atomic t-scale - to most people the only natural one since they don't perceive the atoms of their bodies to be shrinking - is a natural reference frame. Within this frame all fundamental particles and their associated clocks are equivalent.

The position of an accidental particle Q is definable, relative to an arbitrary observer O, by the position of that particle $\mathrm{P} = \mathrm{P}_{FP}$ with which it momentarily coincides, just as its velocity is definable by the velocity of that particle $\mathrm{P}' = \mathrm{P}'_{FP}$ relative to which it is momentarily at rest. Now Q, on account of its motion relative to P, must possess a certain amount of kinetic energy. Shifting our point of view from choosing $\mathrm{O} = \mathrm{P}$ to choosing $\mathrm{O} = \mathrm{P}'$ this energy cannot vanish, all fundamental observers being equivalent; only it is no longer kinetic, but dynamic (potential). So, to Q, it is as if the gravitational potential of the whole universe were centered in P'!

Mogens True Wegener

APPENDIX

Four Models of the Universe

Taking time to be differentiable,
and assuming the RW-metric to hold:

$$d\mathcal{T}^2 = dt^2 - \mathcal{R}^2(t)\,d\sigma^2 = \{dT^2 - d\sigma^2\}\mathcal{S}^2(T)$$

where $\mathcal{R}(t) \equiv dt/dT \equiv \mathcal{S}(T)$, we have
four worlds ***RW*** with the calibrations

$$\delta = 0\,.\Rightarrow.\,t = 0 \Leftrightarrow T = 0$$
$$\delta = 1\,.\Rightarrow.\,t = t_o \Leftrightarrow T = t_o$$

viz. these, cf. Wegener [2007]:

***RW0:* Uniform expansion**

$$\mathcal{R}(t) \equiv 1{+}t/t_o{-}\delta \equiv exp(T/t_o{-}\delta) \equiv \mathcal{S}(T)$$
$$T/t_o = ln(1{+}t/t_o{-}\delta){+}\delta\,.\Rightarrow.\,dT = dt/(1{+}t/t_o{-}\delta)$$
$$d\mathcal{T}^2 = dt^2 - (1{+}t/t_o{-}\delta)^2 d\sigma^2 = \{dT^2{-}d\sigma^2\}exp\,2(T/t_o{-}\delta)$$

***RW1:* Exponential expansion**

$$\mathcal{R}(t) \equiv exp(t/t_o{-}\delta) \equiv 1/(1{-}T/t_o{+}\delta) \equiv \mathcal{S}(T)$$
$$T/t_o = 1{+}\delta{-}exp(\delta{-}t/t_o)\,.\Rightarrow.\,dT = dt/exp(t/t_o{-}\delta)$$
$$d\mathcal{T}^2 = dt^2 - exp\,2(t/t_o{-}\delta)\,d\sigma^2 = \{dT^2{-}d\sigma^2\}/(1{-}T/t_o{+}\delta)^2$$

***RW2:* Hyperbolic sinoidal expansion**

$$\mathcal{R}(t) \equiv sh(1{+}t/t_o{-}\delta)/sh1 \equiv -1/sh\{ln(th\tfrac{1}{2}e^{T/t_o-\delta})\}sh1 \equiv \mathcal{S}(T)$$
$$T/t_o = \delta + ln\{th\tfrac{1}{2}(1{+}t/t_o{-}\delta)/th\tfrac{1}{2}\}\,.\Rightarrow.\,dT = dt\,sh1/sh(1{+}t/t_o{-}\delta)$$
$$d\mathcal{T}^2 = dt^2 - sh^2(1{+}t/t_o{-}\delta)\,d\sigma^2/sh^2 1 = \{dT^2{-}d\sigma^2\}/sh^2\{ln(th\tfrac{1}{2}e^{T/t_o-\delta})\}\,sh^2 1$$

***RW3:* Hyperbolic co-sinoidal expansion**

$$\mathcal{R}(t) \equiv ch(t/t_o{-}\delta) \equiv 1/cos(T/t_o{-}\delta) \equiv \mathcal{S}(T)$$
$$T/t_o = \delta + arsin\{th(t/t_o{-}\delta)\}\,.\Rightarrow.\,dT = dt/ch(t/t_o{-}\delta)$$
$$d\mathcal{T}^2 = dt^2 - ch^2(t/t_o{-}\delta)\,d\sigma^2 = \{dT^2{-}d\sigma^2\}/cos^2(T/t_o{-}\delta)$$

But, after all, why take the RW-metric to be valid?
That GTR presupposes the curvature of space to be constant
is in itself no good reason to accept it if other possibilities are open.
The universe might be locally homogeneous without being globally so.
It might be everywhere isotropic and its curvature locally zero, yet
the curvature might be increasing outwards with distance.
However, the relationship dt/dT is decisive.

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